I used this PDA to work on two subjects that I plan to incorporate in a book on inverse theory that I am working on. As a way to have my results evaluated by peers, I wrote two articles for submission to journals. I already submitted one of them and now I am working on a revision to satisfy some recommendations made by the reviewers regarding presentation. The other article is essentially ready for submission. Below I include the corresponding abstracts.

1) Tutorial: The Backus-Gilbert method and their minimum-norm solution

Abstract:
This tutorial has two objectives. One is to go over the Backus and Gilbert method. Backus and Gilbert investigated continuous inverse problems in the context of a radially symmetric earth using an approach completely different from that of standard inversion methods. If m is an earth property of interest, the basic tenet of the Backus-Gilbert method is that one can only infer an average value of m over some radial interval (w). If w is relatively short and centered near a particular radius r0, the Backus-Gilbert average will be a local average of m near r0 with a resolving length of w. If w is large, the average value of the property cannot be said to be localized near r0. In addition to this concept of resolution, Backus and Gilbert also introduced the concept of tradeoff between resolution and error, which is essential in inverse theory regardless of the inversion method used. In spite of its importance, the Backus-Gilbert method is not described in full in current inverse theory books, which also do not show how to apply the full formalism of the method. In this tutorial I go through the full derivation of the method for linear problems, and apply it to two 1-D problems. The second objective is to discuss the minimum-norm solution for the continuous case, which was also introduced by Backus and Gilbert. Their derivation is little known, and is completely different from a better-known simpler derivation, due to Gilbert. The two derivations are presented here. The minimum-norm solution lacks a constraint that the Backus-Gilbert solution has, which may lead to results affected by significant error, as shown by one of the examples. To address this problem, a constrained minimum-norm solution is derived here, which in this particular example reduces the error considerably.

2) Fitting a straight line to data with correlated errors in both variables.

Abstract:
In a landmark paper, York (Can. J. Phys., 1966) introduced an iterative method for the solution of the problem of fitting a straight line y = ax + b to data with uncorrelated errors in both variables under very general conditions, and later extended the method to correlated errors.
York’s method can be interpreted in terms of the minimization of the sum of the squares of the statistical (or Mahalanobis) distances from the observed points to the best-fit line. Using an approach introduced by Williamson (Can. J. Phys., 1968), this sum can be written as a nonlinear function of \( a \) and \( b \), which here are determined using a standard nonlinear parameter estimation approach. This method is very simple and at least as effective as the York method, as shown by its application to several data sets taken from the literature. One of them is very challenging for existing methods, although not for the one presented here. Because the sum of distances squared is invariant under an interchange of \( x \) and \( y \), using either of them as independent variable should lead to the same best-fit line. Relations derived from this result are used to check the correctness of the solution. As another check, the computed value of \( b \) is compared to the corresponding value derived analytically.