Abstract—Attention plays a crucial role in higher cognition in the presence of limited resources. We study attentional mechanisms producing sequential switching between interacting mental modalities. Our model uses dissipative dynamical systems of coupled oscillators modeling various parts of the brain. The dynamics can manifest winnerless competition (WLC) between the oscillators with switching between different modalities. Heteroclinic cycle is a widely used mathematical image of dynamical switching in WLC. We use the generalized Lotka-Volterra (LV) equations to describe networks with WLC. For simplicity, we assume that there are two subsystems, each is described by three LV equations. Under certain value of mutual coupling one part of the system has topologically equivalent behavior to its initial however the other part exhibits chaotic dynamics. We study the dynamics of the system with intrinsic additive and multiplicative noise components. The coupled attentional system exhibits robust behavior with heteroclinic cycles in the case of multiplicative noise. Additive noise, on the other hand, may lead to the collapse of the complex chaotic dynamics to periodic oscillations.

I. INTRODUCTION

During the past decades, various approaches have been considered for modeling brain and its cognitive functions [1]-[4]. Various classical nonlinear dynamic approaches have also been applied to model attention control dynamics [5], [6]. Winnerless competition (WLC) is a general mathematical principle, which has been shown to play a crucial role in modeling higher cognitive processes [5], [7]. WLC models the dynamics of interacting populations fighting for limited resources. When applying WLC for modeling attention in human brains, the competition involves various areas in the frontal and parietal lobes, such as prefrontal cortex (PFC), anterior cingular cortex (ACC), and basal ganglia (BG) [8], [9]. These areas are responsible for the brain’s executive functions, for learning and reasoning. Prefrontal areas are also concerned with our behavioral patterns, how we interact with the environment, and how our individual personal characters are formed.

In this work we study the interrelation between high level attention and basic perceptual and sensory modalities. First we describe a general framework of attentional cross-modality control [6]. Then we focus on the high-level attentional system. Generalizing previous results, we introduce a coupled system of two interacting nonlinear systems modeled by LV equations. We consider a coupled system where each unit has WLC. The coupled LV equations can be used to describe various dynamic processes in the overall system. In particular, with well-tuned parameters, we can generate heteroclinic cycles. Note that the applied quadratic nonlinearity is the simplest way to introduce complexity to our analysis. We introduce results with various levels of coupling between the subsystems, showing a wide range of dynamical regimes, including the co-existence of heteroclinic cycles and chaos. Next we study the role of multiplicative and additive noise. Finally we discuss on the relevance of the obtained results for attentional switching and control.

II. ATTENTION-PERCEPTION MODEL

The applied attention and perception model includes a hierarchical structure with two levels. The top level represents dynamics of attentional modes. The bottom level of the model describes the behavior of cognitive/perceptual modalities, see Fig.1. The deterministic dynamics of the high-level attentional modes is described by a set of LV equations as follows

$$\frac{dR^{(\ell)}}{dt} = R^{(\ell)} \left( \gamma^{(\ell)} - \sum_{k=1}^{N} a_{\ell,k} R^{(k)} \right), \quad (1)$$

where $R^{(\ell)}$ is the $\ell$-th attentional mode, $\ell = 1, \ldots, N$. The total number of interacting modes $N$ describes the metastable composition of elements from different brain areas that are inter-related to perform a specific cognitive task. Time constants $\tau_\ell$, are fixed for a LV system, and parameters $\gamma^{(\ell,k)}$ and $a_{\ell,k}$ describe the inhibitory connections. The lower-level of the hierarchy modeling the perceptual/sensory modalities is described by the following LV equations

$$\frac{dX^{(\ell)}}{dt} = X^{(\ell)} \left( \sigma_{l,t} - \sum_{j=1}^{N_i} p_{l,j} X_{j}^{(l)} \right) - \sum_{k=1}^{M} \sum_{l,k \neq l}^{N_k} c_{i,k}^{l,k} X_{l}^{(k)} \left(1 \right) .$$

where the time constants are $\theta_{\ell}$, and coupling parameters $\sigma_{l,t}$, $p_{l,j}$, and $c_{i,k}^{l,k}$ describe the inhibitory connections between perceptual modalities $X_{i}^{l(\ell)}$.
III. NOISE EFFECTS IN THE ATTENTIONAL MODEL

The generalized Lotka-Volterra equations are suitable model to describe heteroclinic cycles and to model synchronization in competitive networks. However, in real application including modeling brain activity, we need to take into account the presence of noise. The presence of noise may be included in a model in two different ways, such as additive or multiplicative. The influence of each type of noise on the behavior of the system are, in general, different. Let us focus on the attentional (top level) system and investigate the influence of noise in interacting modes. Thus, we consider the following model

\[ \frac{dR^{(l)}}{dt} = R^{(l)} \left( \gamma_l - \sum_{k=1}^{N} (a_{l,k} + \epsilon_{l,k}(t)) R^{(k)} \right) + \eta_l(t), \]  

where \( l = 1, \ldots, N \). Here \( R^{(l)} \) describes the \( l \)-th node of the system and \( a_{l,k} \) is the inhibitory connection. In this paper we consider the case when the total number of nodes is \( N = 6 \). We used Runge-Kutta ODE solver of order 4 with fixed time step. In our present studies we use time step 0.005.

Let the subsystem \( S1 \) which consists of \( R^{(l)} \) with \( l = 1, 3, 5 \) be the odd subsystem; similarly we can define \( S2 \) to be an even one. We assume that time constants \( \tau_l \) are the same in a given subsystem. \( \{a_{l,k}\} \) for \( k = l \pm 2(mod6) \) is connection matrix for a subsystem and \( a_{l,l} = 1 \) for all \( l \). The coupling between the two subsystems is described by the connection coefficient \( \kappa = a_{l,l+1} \) for \( l = 1, \ldots, 6 \) and subindex is taken \( (mod6) \). The other parameters are set to zero. \( \epsilon_{l,k} \) and \( \eta_l \) are derived from Gaussian noise components. We consider the absolute value of additive noise to guarantee that all \( R^l \) remain positive. Similarly, we take absolute value of multiplicative noise to make sure that the coupling always remains inhibitory. We may tune parameters of both subsystems such that they exhibit heteroclinic cycle while coupling parameter \( \kappa = 0 \).

We choose parameters \( \gamma_l \) and \( \omega_{l,k} \) to guarantee existence heteroclinic cycles [10], [11] and in present study they are \( \gamma_1 = 1.5, \gamma_3 = 1.426, \gamma_5 = 0.956, \gamma_2 = 0.8, \gamma_4 = 1.28, \gamma_6 = 2.5 \).

1.4.: For the other coupling parameters, see [6]. Time constants are \( \tau_{odd} = 1 \), and \( \tau_{even} = 1.7 \). These internal parameters will not be changed in the present studies.

We analyzed the behavior of the system with the different magnitude of coupling parameter \( \kappa \). Figure 3. In this case we selected each systems so they exhibit stable heteroclinic cycles
at which level of noise the heteroclinic and chaotic trajectories will be finally destroyed.

IV. Conclusion

Our present work is related to extensive studies of attentional mechanism in humans, in particular to the topic of the experimentally observed sequential switching between attentional modes. The duration of a metastable attentional mode is measured to be about 0.1-0.2 s, i.e., the switches occur at the theta frequency band. The described model is capable of describing the multi-level hierarchy of attentional and perceptual/sensory interaction, however, here we focus on the high-level attentional system.

- Generalizing previous results, we introduce a coupled system of two interacting nonlinear systems modeled by LV equations. We consider a coupled system where each unit has WLC. We introduce results with various levels of coupling between the subsystems, showing a wide range of dynamical regimes, including the coexistence of heteroclinic cycles and chaos. We study both deterministic and noisy systems.

- In the deterministic case, the WLC dynamics, when coupling parameter $\kappa > 0$, we get the closed composition system $S_1 + S_2$ that has higher dimensional phase space with bigger number of fixed points. This implies that a heteroclinic cycle may lose its robust property. For example, if coupling parameter $\kappa = 0.011$, one subsystem $S_1$ still has heteroclinic cycle, while $S_2$ becomes chaotic.

- Properly selected additive noise level can destroy the heteroclinic cycle and lead to the emergence of limit cycle in its vicinity. It represents the narrow-band rhythmic activity of the system. This phenomenon resembles a resonance, and further studies are needed to evaluate the range and validity of such resonance effects. It may be related to pathological brain conditions caused by the destruction of sequential switching describing normal operation of attentional process.

- Multiplicative noise does not destroy the heteroclinic cycle, but it gets increasingly noisy. This means that the heteroclinic cycles are structurally stable in the presence of multiplicative noise as applied in our model. The duration of the heteroclinic cycle increases with increasing multiplicative noise levels and it is expected that above a threshold level of noise the sequential switching ceases. This result is very useful in future implementation of the model for attentional crossmodality control with perceptual hierarchy.

References


