Dynamic Logic Applied to SAR Data for Parameter Estimation Behind Walls

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Abstract—Identifying and localizing targets within buildings using exterior sensors will offer superior advantages to the military and law enforcement communities. Research on wall-penetrating radar has produced significant advances in recent years regarding this topic. However, wall parameter ambiguities, multiple reflections, clutter, and measurement noise pose significant challenges to developing robust detection and estimation methods. In the present work we demonstrate can be mitigated using dynamic logic (DL), an adaptive method for iterative maximum likelihood.

I. INTRODUCTION

Transmitting and receiving RF energy through the exterior wall of a building distorts signals reflected from interior targets in several ways, even if the wall is composed of lossless material. If weighting applied to the received data in the estimation process does not compensate for the slower propagation time, refraction and attenuation through the wall, targets will appear delayed, smeared (defocused) and lower in magnitude [1]. The severity of these effects depends on the wall thickness and dielectric constant. For a synthetically produced linear array, the degrees of refraction and delay vary with the look angle at each transmit/receive position. If compensation for the exterior wall is employed, the image will remain ill-defined unless the thickness and dielectric constant are known exactly [2]. Autofocusing techniques using image quality feedback have shown that accurate estimation of one of the wall parameters is possible if the other is known [3][2]. Estimating both thickness and permittivity may result in a number of local maxima where erroneous combinations of wall parameters could still provide reasonable focusing [3]. These techniques, however, rely on the computational burden of processing an image, require a reference target behind the wall to compute focusing metrics, or require a complex multiple input, multiple output (MIMO) sensor arrangement [4].

We address the problem of estimating the thickness, \(d_1\), and dielectric constant, \(\varepsilon_1\), of the exterior wall using the unprocessed, complex data received over the synthetic array observation. Additionally, we do not assume the target is known but rather attempt to concurrently estimate its cross-range and range position \((x_t, y_t)\). Our model-based algorithm using DL [5] iteratively determines the four unknown parameters \(d_1, \varepsilon_1, x_t, \) and \(y_t\) without the combinatorial complexity associated with other model-based approaches such as multiple hypothesis testing. Furthermore, the technique has the potential capability to overcome the additional burdens of receiver noise, multipath versions of the target, and clutter from unknown objects.

Standard synthetic aperture radar (SAR) image processing is not reliable due to the environmental complexity and the compound effects the exterior wall and inner walls have on scattering within the structure. A model-based approach to the problem may be the only way to really determine the location and intensity of objects and interior features. The challenge of “seeing” into buildings is highlighted by the simple experiment detailed in this paper. The unknown parameters are so highly coupled that the maximum likelihood solution is virtually indistinguishable from many other likelihood values based on incorrect estimate combinations. Even in the noiseless case an iterative estimation method is likely to converge on a false solution and is highly dependent on the initial guesses. Additive measurement noise can completely distort the multi-dimensional likelihood surface that we are attempting to maximize. Several innovations are examined in this paper that could possibly mitigate these complications. In particular, Section III describes how Fisher information [6] offers insight to partially decoupling the wall parameters and thus facilitating their estimation.

Alternatively, we could compute the entire likelihood function for the four-dimensional parameter space, and select the parameters that maximize the function. This approach would yield the same intended result as DL but is much more computationally intensive, even for the relatively simple four parameter problem. The number of computations would increase exponentially as the unknown parameter set increases.
and would not necessarily provide an accurate result due to coupling and interference (noise and clutter). Adapting to a realistic situation where thousands of parameters need to be estimated, only an iterative, model-based approach such as DL is feasible given the linear increase in computations of this method with additional models and parameter estimates.

II. SIGNAL MODEL

Our approach to wall penetrating SAR minimizes the complexity of the physical system by modeling only the outer wall. A single point scatterer is located at least several wavelengths behind a wall of infinite length and thickness $d_1$. The scene is defined by a two-dimensional Cartesian coordinate system where the cross-range and range directions, relative to the radar, are represented by $x$ and $y$ respectively. The radar is located in front of the wall, transmitting and receiving in the far field at various array positions indexed by $u_1$, $u_2$, ..., $u_{N_u}$ along a linear path that is parallel with the wall. Figure 1 depicts the geometry for the radar at array position $u$ as it moves in the $+x$ direction, and its association with the target located $x_t$, $y_t$. The wall material is non-ferromagnetic and lossless with relative permittivity $\varepsilon_1$. Using these simplifications, the wall boundary transmits a portion of the incident field from the radar at a refracted angle $\theta_1$ through the wall, and reflects the residual energy in the specular direction. The ray tracing assumption through a wall of infinite length discounts the direct effects of the wall reflection on the receiving antenna. The displacing and defocusing effect the wall has on the target will be seen is the wall media.

The SAR signal model as a function of spatial sample $u$ and frequency sample $k$ given a single point scatterer in free space is [7]

$$s_{u,k} = \frac{|a_t|}{R_t(u)} e^{j(\phi_0 - \beta_k 2R_t(u))},$$

where the complex reflectivity of the target is $a_t$, with magnitude $|a_t|$ and phase $\phi_0$, and $R_t(u)$ is the range from the radar at position $u$ to the target. The wave number $\beta_k = \omega_k/c$, where $c$ is the speed of light and $\omega$, $k = 1 \ldots N_k$, are the angular frequency samples of the linear FM pulse. Although in practice $a_t$ is frequency and aspect angle dependent, we are assuming constant, isotropic scattering.

The signal model used for SAR is altered based on the simplified physical scene in Figure 1. We examine electric fields in the general case of a perpendicularly polarized wave propagating at an oblique angle through a lossless material. Using Snell’s Law, the angle of refraction $\theta_1$ is related to the angle of incidence $\theta_0$,

$$\beta_{0k} \sin \theta_0 = \beta_{1k} \sin \theta_1,$$

where $\beta_{0k}$ is the wave number for free space and $\beta_{1k}$ is the wave number modified by the velocity of propagation through the wall media.

Referring to the diagram in Figure 1 and the nomenclature used for SAR in free space, the received signal model proceeds from the electromagnetic field definitions as follows,

$$s_{u,k} = \frac{|a_t|}{R_t(u)} \frac{T_{01}(u)T_{12}(u)T_{21}(u)T_{10}(u)}{[R_0(u) \cdot R_1(u) \cdot R_2(u)]^2} \times e^{j(\phi_0 - 2\beta_{0k} R_0(u) - 2\beta_{1k} R_1(u) - 2\beta_{0k} R_2(u))}$$

where the $T_{01}(u)$ is the transmission coefficient from region 0 (the radar) through region 1 (the wall), $T_{12}(u)$ is the transmission coefficient from region 1 through region 2 (the target), and $T_{21}(u)$ and $T_{10}(u)$ are the backscatter transmission coefficients. These values are angle dependent and thus vary with the radar and target positions.

Since we assume free space in regions 0 and 1, and the wall has permeability $\mu_0$, we can write [8]

$$T_{01}(u) = \frac{2 \cos \theta_0(u)}{\cos \theta_0(u) + \sqrt{\varepsilon_1} \cos \theta_1(u)} = T_{21}(u)$$

$$T_{12}(u) = \frac{2 \cos \theta_1(u)}{\cos \theta_1(u) + 1/\sqrt{\varepsilon_1} \cos \theta_0(u)} = T_{10}(u)$$

$$T_{12}(u) = \frac{\cos \theta_1(u)}{\cos \theta_0(u) \sqrt{\varepsilon_1} T_{01}(u)}.$$

Thus the received signal model simplifies to

$$s(u,k) = \frac{|a_t| T_{01}^*}{[R_0(u) \cdot R_1(u) \cdot R_2(u)]^2} \cos^2 \theta_1(u) \times \varepsilon_1 e^{j(\phi_0 - 2\beta_{0k} R_0(u) + 2\theta_1(u) - 2\beta_{0k} R_2(u))}.$$

Attempting to solve for the angles $\cos \theta_1(1) \ldots \cos \theta_1(N_u)$
along the array results in a transcendental equation. Thus we determine them numerically using Newton’s method, as outlined in [9].

III. THE DYNAMIC LOGIC ALGORITHM

A. Background

The model estimation algorithm presented below is based on DL [5]. Let \( \Theta_m \) denote the set of unknown parameters to be estimated for model \( m \), and set \( \Theta = [\Theta_1^T \ldots \Theta_M^T]^T \). Further, we define \( r = [r_{1,1} \ r_{1,2} \ldots]^T \). In this work we are concerned with two models, \( m = 0 \): interference (noise/clutter) and \( m = 1 \): the scene (exterior wall with a point scatterer behind it). In general there is no restriction on the number of models included in the algorithm. For instance we could hypothesize there are several targets behind the wall, or there are multiple interior walls, or there is a staircase in the room, etc. Dynamic logic simultaneously compares any number of models to the data and iteratively updates the parameter estimates for each model. Thus it requires only a linear increase in computations as the number of models and/or parameter estimates increase. This yields a significant advantage over other model-based algorithms that require an exponential increase in computations.

The system consists of mixture components or classes, each characterized by its own probability density function (pdf). In general, the goodness of the fit between a model and data is often described by the following form of the joint likelihood function:

\[
L = \prod_{m=0}^{M-1} \prod_{u=1}^{N_u} \prod_{k=1}^{N_k} p(r_{u,k}|\Theta),
\]

where \( p(r_{u,k}|\Theta) \) is the probability density of the data conditioned on the parameter set \( \Theta \). Under each model, the observations \( \{r_{u,k}\} \) are assumed to be independent, as shown in (6).

The DL algorithm is an iterative process that maximizes the mixture of \( M \) likelihood functions

\[
L = \sum_{m=0}^{M-1} \hat{\alpha}_m \prod_{u=1}^{N_u} \prod_{k=1}^{N_k} p(r_{u,k}|\Theta),
\]

subject to the constraint that \( \sum_{m=0}^{M-1} \alpha_m = 1 \), where \( \alpha_m \) is the prior probability associated with each model. The hats on \( \alpha_m \) and \( \Theta \) indicate the priors and the model parameters are estimated. The algorithm begins with initial guesses of \( \Theta_m \) and \( \alpha_m \), and computes a similarity measure between the data and the models based on these guesses. The initial model variances are large, indicating a “fuzzy” relationship with the data. As the algorithm iterates, estimates for \( \Theta \) and \( \{\alpha_m\} \) are updated, and the variances are reduced (usually based on an exponential decay) so that subsequent relationships are less fuzzy. The update equation for \( \alpha_m^{i+1} \) is given by

\[
\alpha_m^{i+1} = P(m|r) = \frac{\alpha_m p_m(r | \Theta_m^{i+1})}{\sum_{m=0}^{M-1} \alpha_m p_m(r | \Theta_m^{i+1})},
\]

where the normalization condition \( \sum_{m=0}^{M-1} \alpha_m = 1 \) is satisfied.

As shown from this equation the \( a \ priori \) probability estimates are set to equal the current \( a \ posteriori \) probabilities. The updates \( \Theta^{i+1} \) are determined by gradient-based approximations to the maximum likelihood estimators for the individual models. The process terminates when the similarity measure no longer changes for succeeding iterations and estimates converge to the maximum likelihood solution. At this point the variances are small suggesting a “crisp” final relationship between the models and data.

B. Adaptation to Target/Wall Estimation

For the current work we assume a two-dimensional scene including the radar, wall and target in the same plane as illustrated in Figure 1. The DL algorithm is applied to the data after the observation is carried out and before an image is processed (prior to any phase corrections). The set of available data is given by \( r_{u,k} = s_{u,k} + n_{u,k} \), where the term \( n_{u,k} \) is comprised of independent and identically distributed complex circular Gaussian random variables. The pdfs incorporate a propagation model which predicts expected signal values of \( r_{u,k} \) for unknown values of the parameter set. The pdf for class \( m = 0 \) (noise) is Gaussian for a complex variable,

\[
p_0(r | \Theta) = \frac{1}{\pi \sigma_0^2} e^{-\frac{1}{\sigma_0^2}(r_{u,k} - \hat{\Theta}_{0})^2},
\]

for which \( \Theta_0 = 0 \) and the pdf for class \( m = 1 \) (scene) is,

\[
p_1(r | \Theta) = \frac{1}{\pi \sigma_1^2} e^{-\frac{1}{\sigma_1^2}(r_{u,k} - \hat{\Theta}_{1})^2},
\]

where \( I_0(\cdot) \) is the modified Bessel function of the first kind of order zero. Note the argument in \( I_0 \) is the magnitude of the correlation function between the data \( r_{u,k} \) and the model \( s_{u,k}(\hat{\Theta}) \) given the current estimate of \( \Theta \). Here, \( s_{u,k}(\hat{\Theta}) \) is the signal model estimate of data component \( \{u, k\} \) given the parameter set \( \Theta \). Note that for \( p_0(r_{u,k} | \Theta) \), \( \sigma_0^2 \) is the variance of each in-phase and quadrature white noise term. However, for \( p_1(r_{u,k} | \Theta) \), \( \sigma_1^2 \) is the mean squared error between the model and the data. If \( \hat{\Theta} = \Theta \) then \( \sigma_1^2 \) equals the noise power.

The task of the DL algorithm is to distinguish between the noise and scene models, and estimate the parameter set \( \Theta = [x_t \ y_t \ d_1 \ \varepsilon_1] \), that is the target position in azimuth and range, the wall thickness and the dielectric constant. The flow chart in Figure 2 summarizes the process for wall and interior target estimation. For iteration \( i \), estimates for \( \hat{\Theta}^{(i)} \) are fed into an electromagnetic modeling program. In our case we analytically predict the signal model \( s_{u,k}(\hat{\Theta}^{(i)}) \) based on Figure 1, but in practice an efficient computational EM method is needed. A mixture model is composed using \( s_{u,k}(\hat{\Theta}^{(i)}) \), \( \alpha_m^{(i)} \) and \( \sigma_1^{(i)} \), which is then used to update the parameter sets and mixture parameters. The new signal model \( s_{u,k}(\hat{\Theta}^{(i+1)}) \) is tested for
convergence and feedback along with $\alpha_m^{(i+1)}$ and $\sigma_2^{(i+1)}$ to form a new mixture model.

\begin{align*}
\text{Initialize parameters} & \quad \Theta_m^{(0)} \quad \alpha_m^{(0)} \quad \sigma_2^{(0)} \\
\text{Compute electromagnetic models} & \quad s_{u,k}(\Theta_m^{(0)}) \\
\text{Sensor data} & \quad r_{u,k} \\
\text{Compute mixture models for iteration (i)} & \quad \sum_m \alpha_m^{(i)} p(r|\Theta_m^{(i)}) \\
\text{Update model and mixture parameters} & \quad \Theta_m^{(i+1)} \quad \alpha_m^{(i+1)} \quad \sigma_2^{(i+1)} \\
\text{Final parameter Estimates} & \quad \Theta_m^{(i+1)} \\
\text{Test for convergence} & \\
\text{Compute electromagnetic models} & \quad s_{u,k}(\Theta_m^{(i+1)})
\end{align*}

Fig. 2. Summary of the dynamic logic estimation process

An estimation method based on DL requires a similarity measure, or objective function that must be clearly defined and robust, in the sense that local maxima can be avoided. Maximizing the mixture model (7) is equivalent to maximizing its logarithm

$$
\ell = \ln [\alpha_0 p_0 + \alpha_1 p_1].
$$

Updating the estimates in $\Theta$ requires computing the partial derivative of (11) with respect to each parameter. Below we detail the development for updating the generic parameter $\theta$, which can be $x_t$, $y_t$, $d_1$, or $\varepsilon_1$. Here we vectorize the signal model as $s(\Theta)$. Noting that the variable only appears in the signal model,

$$
\frac{\partial \ell}{\partial \theta} = \frac{1}{[\alpha_0 p_0 + \alpha_1 p_1]} \frac{\partial}{\partial \theta} \alpha_1 p_1
$$

$$
= \frac{\alpha_1 p_1}{[\alpha_0 p_0 + \alpha_1 p_1]} \frac{\partial}{\partial \alpha_1} \alpha_1 p_1 = P(1| r_{u,k}) \cdot \frac{\partial}{\partial \ln p_1}
$$

$$
= P(1| r_{u,k}) \text{ } \partial \frac{\partial}{\partial \theta} \left[ -\frac{1}{\sigma_i^2} \|s(\Theta)\|^2 + \ln I_0 \left( \frac{2}{\sigma_i^2} \right) \right]
$$

(12)

where $r^H$ is the conjugate transpose of the data vector. Defining the function $v = \|r^H s(\Theta)\|^2 + \ln I_0 \left( \frac{2}{\sigma_i^2} \right)$, the derivation continues

$$
\frac{\partial \ell}{\partial \theta} = -P(1| r_{u,k}) \frac{2}{\sigma_i^2} \left[ \text{Re} \left\{ s^H(\Theta) s'(\Theta) \right\} + v' \right]
$$

(13)

where $s'(\Theta)$ and $v'$ are the partial derivatives of the signal model and the correlation magnitude with respect to $\theta$, which can be computed analytically or numerically. $I_1(\cdot)$ is the modified Bessel function of the first kind of order one.

C. Fisher Information for Wall Parameter Estimation

The authors in [4] state that gradient search techniques to minimize the error in wall ambiguities are not applicable. Given that $d_1$ and $\varepsilon_1$ are highly coupled, their concurrent estimation is nearly impossible using any technique. However, expressions that define the Fisher information matrix for their joint estimation offer insight to partial decoupling which mitigates the problem.

By defining the two diagonal matrices defining the matrix

$$
\mathbf{Y} = \text{diag} \left[ \beta_{0,1} R_1(1) \cdots \beta_{0,N_k} R_1(1) \beta_{0,1} R_1(2) \cdots \beta_{0,N_k} R_1(N_u) \right],
$$

and $\Phi = \text{diag} [\beta_{0,1} R_1(1) \cdots \beta_{0,N_k} R_1(1) \beta_{0,1} R_1(2) \cdots \beta_{0,N_k} R_1(N_u)]$, the Fisher information matrix is assembled as such,

$$
\mathbf{J} = \begin{bmatrix}
\frac{2}{\varepsilon_1 \sigma_0^2} s^H \mathbf{F} \mathbf{F} s & \frac{4 d_1}{\sigma_0^2} s^H \mathbf{F} \mathbf{Y} s \\
\frac{4 d_1}{\sigma_0^2} s^H \mathbf{Y} s & \frac{8 d_1^2 \varepsilon_1}{\sigma_0^2} s^H \mathbf{Y} \mathbf{Y} s
\end{bmatrix}
$$

(16)

and the determinant is

$$
\text{det}(\mathbf{J}) = \frac{16 d_1^2}{\sigma_0^2} \left[ (s^H \mathbf{F} \mathbf{F} s)(s^H \mathbf{Y} \mathbf{Y} s) - (s^H \mathbf{F} \mathbf{Y} s)^2 \right].
$$

(17)

Here the signal model does not depend on the estimated parameter set $\hat{\Theta}$ but rather the true values.

The theoretical lower bounds on the error variances of $d_1$ and $\varepsilon_1$, i.e., the Cramér Rao bounds are computed by inverting $\mathbf{J}$. However, $\mathbf{J}$ is poorly conditioned if the factor in the brackets on the righthand side of (17) is close to zero. That depends on the variation in range though the wall from the different array positions, $R_1(1), R_1(2), \ldots, R_1(N_u)$. In fact if the radar collects all its data from one position, then the range vector through the wall is constant and $\mathbf{J}$ becomes singular. Moving the synthetic array closer to the exterior wall or including vertical excursion in the observation path are potential ways to increase the angle diversity through the wall, which subsequently conditions the Fisher information matrix and partially decouples the wall ambiguities.

D. Two-stage Processing and Variable Weighting

The DL algorithm has shown to be accurate and robust in simulations if three out of the four parameters are known. The multi-variable problem is much more challenging since the parameters are so highly coupled. However, the cross-range location of the target is far less coupled than the remaining unknown parameters are to each other. The azimuth resolution gained by the large synthetic aperture allows for reliable scattering estimation in the cross-range “strips” of a SAR image, even when the fields propagate through the wall. However, under conditions of low SNR and/or a high amount of clutter
from interior walls and objects, an iterative estimation process is required to determine the target cross-range position. In our simulated experiment we divide the estimation process into two stages. In the first stage, initial guesses of the unknown parameters are made and DL algorithm is used to estimate \( x_t \) only, since incorrect values of \( d_1, \varepsilon_1, \) and \( y_t \) do not drastically influence the result. With the assumption that \( \hat{x}_t \) is fairly accurate, the dimensionality of the problem is reduced to three unknown parameters, which are concurrently estimated using DL in the second stage.

Since \( \nu \) in (13) comprises both \( r \) and \( s(\Theta) \), it is the term that dominates in the estimation process. For every iteration \( i \) the partial derivatives of correlation between the data and the signal model are computed given the current parameter estimates \( \Theta^{(i)} \). If we assume for the moment that there is only one unknown parameter, for instance the target range \( y_t \), then the behavior of the objective function would be “tightly” located around the true value, assuming the signal to noise ratio (SNR) is relatively large. In this case, if the initial guess at the range estimate is already within the mainlobe of the objective function then the solution will converge to the peak and result in a final estimate that is very close to the true value. If the initial guess is outside of the mainlobe, then the risk of maximizing to a sidelobe value is substantial. For low SNR, the mainlobe will not be well defined and there is an even greater probability that the solution will converge to a sidelobe maximum. Furthermore, when multiple parameters are unknown, the objective functions of each parameter are not necessarily centered around the true values due to their coupled nature, even for large SNR. For example, if the initial guess of the wall thickness is larger than \( d_1 \), the electromagnetic model assumes a larger delay than necessary and the objective function for \( y_t \) will peak at a greater range than the true target. If the algorithm converges too quickly it will maximize to an erroneous value of \( \hat{y}_t \), which will influence the estimates for \( d_1 \), and \( \varepsilon_1 \).

To mitigate this problem, the objective functions at the start of the algorithm need to be “fuzzy”, or broadly placed around the maximizing value and exhibit negligible sidelobes. For subsequent iterations the functions should become increasingly tighter. This can be achieved by weighting the spatial and frequency content of the data and signal model such that a small amount of information is included at the start. The estimation process begins with poor resolution so that the objective functions are broadly located around the true values and the first few updates will be in the correct direction. As the estimates are improved, a gradual increase in the amount of data is included until the final convergence where we exploit the full resolution of the system. The data is organized in a matrix where the rows represent array positions and the columns represent frequency samples of the pulse from each array position. A two-dimensional Gaussian weighting function is applied having small initial variances that increase with iteration at exponential rates based on predetermined time constants. In this way the migration from a “fuzzy” to “crisp” process is accomplished, while in addition, sidelobes in both range and cross-range are minimized.

### IV. SIMULATION

In the following simulated experiment we apply the DL algorithm to simulated data received based on the configuration in Figure 1 with an integrated SNR = 20 dB. The transmitted pulse is centered at 2 GHz with a bandwidth of 1 GHz, and the synthetic aperture length is 25 m at a nominal distance from the target of 50 m. This corresponds to resolutions in azimuth and range of \( \delta_z = \delta_r = 0.15 \text{ m} \). Table IV displays the true parameter values, the initial estimates and the final estimates. Note that the initialized position values are more than one wavelength from the true value. The value \( x_t \) was estimated first and the result was included in the second stage to estimate \( y_t, d_1 \) and \( \varepsilon_1 \).

Figure 3 shows the initial objective function for \( x_t \) in the first stage which is broadly positioned around the true cross-range location, hence the first few \( \hat{x}_t \) updates move in the right direction toward \( x_t \). In figure 4, the objective function becomes more resolved in cross-range but remains unresolved, i.e. fuzzy, in range because the small variance associated with the Gaussian weighting of the bandwidth does not increase while the spatial variance does increase. This way the incorrect initial estimate \( \hat{y}_t^{(0)} \), as well as \( d_1^{(0)} \) and \( \hat{\varepsilon}_1^{(0)} \), do not affect the convergence of \( \hat{x}_t \). The final estimate \( \hat{x}_t^{(f)} \) is used in the second stage of the DL algorithm to estimate the remaining parameters and the results are seen in the table.

Figures 5, 6, 7, and 8 show the updated estimates as a function of iteration. The random noise realization for this example happened to allow almost perfect convergence to the true values. In reality the error variances of the estimates in noise are bounded by the Cramér–Rao inequality. In future publications we plan to provide Monte Carlo analysis of the DL process in order to examine how close the empirical error variances of the estimates come to the corresponding Cramér Rao bounds.

<table>
<thead>
<tr>
<th>( x_t )</th>
<th>( y_t )</th>
<th>( d_1 )</th>
<th>( \varepsilon_1 )</th>
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<tbody>
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<td>0.50 m</td>
<td>5</td>
</tr>
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<td>( \hat{d}_1^{(0)} )</td>
<td>( \hat{\varepsilon}_1^{(0)} )</td>
</tr>
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<td>-0.25 m</td>
<td>0.30 m</td>
<td>3</td>
</tr>
<tr>
<td>( \hat{x}_t^{(f)} )</td>
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<td>( \hat{d}_1^{(f)} )</td>
<td>( \hat{\varepsilon}_1^{(f)} )</td>
</tr>
<tr>
<td>0 m</td>
<td>0.01 m</td>
<td>0.49 m</td>
<td>4.99 m</td>
</tr>
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</table>

### V. CONCLUSION

Some details of the algorithm are excluded in this publication. As noted in the text, convergence is not always accurate even in the noiseless case due to the coupled nature of the parameters. Increasing the angle diversity through the exterior wall can begin to decouple the wall ambiguities but a unique solution can still be elusive. Variable Gaussian weighting on the data is an attempt to slowly update the estimates so that the effects of noise and coupling can be reduced, but they cannot
Fig. 3. The initial, fuzzy 2D objective function is centered around $x_t$ for incorrect initial estimates of $y_t$, $d_1$ and $\varepsilon_1$.

Fig. 4. The 2D objective function retains its orientation around $x_t$ as more data is included and resolution in cross-range is reduced.

Fig. 5. Estimation of $x_t$ by itself with SNR = 20dB and incorrect values of target range and wall parameters. The true value is at 0 m.

Fig. 6. Estimate of $y_t$ with SNR = 20dB. The true value is at 0 m.

Fig. 7. Estimate of $d_1$ with SNR = 20dB. The true value is 0.5 m.

Fig. 8. Estimate of $\varepsilon_1$ with SNR = 20dB. The true value is 5.

be eliminated. Therefore results depend on the initial estimates, as well as the noise realization.

For future research we plan to examine the effect of using a sparse planar synthetic array, where vertical excursion of the sensor is included as the data is collected [11]. This retains the benefit of making a single-pass observation while gaining the angle diversity that is needed to decouple the unknown parameters, and thus provide more reliable convergence.

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