RESEARCH REPORT

Robust Liner Shipping Scheduling: A Game Theoretic Approach

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ABSTRACT
Uncertainty in marine container terminal operations negatively affects reliability of liner shipping services. Unexpected disruptive events may cause significant vessel service delays and in turn delay product delivery to customers. This report proposes a new methodology for modeling uncertainty in port handling times in liner shipping scheduling and presents a bi-level bi-objective mixed integer non-linear mathematical model to minimize the average route service cost and the route service cost range. An evolutionary algorithm is developed to solve the resulting problem. Numerical experiments indicate that the proposed methodology provides more robust vessel schedules as compared to the deterministic approach.
INTRODUCTION

Improving reliability of vessel schedules remains a challenging task for liner shipping companies. Off-schedule vessels substantially disrupt port operations, increase idling time of human operators and equipment, affect terminal gate throughput, storage yard operations, result in bottlenecks, etc. According to Cargo Business (1), “the Asia-Europe trade was the least reliable during August-October (2014) with only 58% of ships arriving on-time”, while reliability of Transpacific and Transatlantic routes averaged on 62% and 77% respectively, which is considered as unacceptable by many shippers. The average difference between actual and scheduled days of arrival at those routes remained 1.1 days for August-October 2014. Such delays can be caused by various factors, including inclement weather, port congestion, slow steaming, or “decline in the carriers’ network operations” (2). Drewry Maritime Research underlined that although vessel sharing with other carriers (or alliance partners) could decrease freight rates, but at the same time could reduce the vessel schedule reliability (2).

Congestion was observed at some European and United States (U.S.) West Coast ports. Vessel arrival delays of 70 hours were reported at major European ports, including Rotterdam, Hamburg, and Antwerp, in July 2014 (3). Some experts indicate that port congestion could be caused not only by peak-season volumes, but also by the inability of container terminal operators to efficiently handle increasing quantities of large size vessels. According to the Journal of Commerce (3), around 50% of Post-Panamax vessels faced more than 12 hour delays at North and South American ports in July 2014. Despite increasing number of uncertainties that may significantly delay vessels, liner shipping companies are trying to improve reliability of their services and design robust vessel schedules. Maersk, the largest liner shipping company in the world, sets a 95% reliability target for its vessels (2).

This report considers the vessel schedule design problem for a liner shipping route, where port handling times are not known with certainty. A new bi-level bi-objective formulation is proposed (i.e., Pareto-Stackelberg Equilibrium), minimizing the average route service cost and the route service cost range. Due to the complexity of the mathematical model, an evolutionary algorithm (EA) is developed and adopted as the solution approach. Numerical experiments were conducted to compare schedules obtained using the proposed methodology and a deterministic approach.

The rest of the paper is organized as follows. The next section reviews the relevant literature, focusing on uncertainty in vessel scheduling. The third section presents the mathematical formulation of the problem, while the fourth section discusses the problems’ complexity and design of the solution algorithm. The fifth section describes numerical experiments, performed in this study, and the last section summarizes findings and outlines future research directions.

LITERATURE REVIEW

The problem of vessel scheduling in a liner shipping route receives constant attention from the research community. Decisions made by a liner shipping company can be divided into three categories (4): a) strategic (long-term), b) tactical (medium-term), c) operational (short-term & real-time). Long-term decisions include vessel fleet size and mix, alliance strategy, and network design, while medium-term decisions focus on service frequency determination, fleet deployment, sailing speed optimization, and construction of the vessel schedule. At the
operational level the liner shipping company has to decide on cargo booking, cargo routing, vessel rescheduling and potential reject of cargo. All decision problems have been studied in depth over the last decade. However, only a few studies considered uncertainty in liner shipping operations. The literature review presented herein is focused on tactical and operational level problems, where vessel sailing speeds and/or port times are not known with certainty.

Chuang et al. developed a fuzzy EA to solve the containership routing problem, taking into account uncertainty in demand, vessel sailing speeds and port handling times. The objective maximized the total profit, estimated as a difference between the total revenue and the total route expenses. The route cost included voyage costs and port handling costs. A fuzzy logic based on the triangular distributions was applied for modeling uncertainty in demand, vessel sailing speeds, and port handling times. Numerical experiments demonstrated efficiency of the proposed methodology and the solution approach. Qi and Song considered the vessel scheduling problem, taking into account the impact of port time uncertainty. The objective minimized the total expected fuel consumption and penalties due to vessel delays. Simulation-based stochastic approximation methods were used to solve the problem. The port times were assumed to follow the uniform distribution. Six scenarios with different levels of port time uncertainty were considered. Computational examples indicated that increasing uncertainty in port times caused greater fuel consumption for a given route. Wang and Meng presented a liner shipping route schedule model, capturing uncertainty in sailing and port times. The objective of a mixed integer non-linear program minimized the total transportation cost. Port time contingency was modeled using the uniform distribution, while uncertain sailing time was estimated based on realization of a port time and an additional parameter, denoting hedge against contingency (proportional to length of a voyage leg). The original program was reformulated as a linear problem and solved using CPLEX. A computational example was provided for the Asia-Europe-Oceania shipping network. It was found that sailing and port time contingency could require deployment of more vessels on a given route.

Wang and Meng studied a liner shipping route scheduling problem, taking into account possible uncertainties in port waiting times (due to congestion) and container handling times. The objective of a mixed integer non-linear program minimized the total transportation cost. Uncertainties in port waiting and service times were modeled using the truncated normal distributions. The original problem was linearized and solved using CPLEX. Sample average approximation (SAA) was used to estimate the expected values for stochastic port waiting and service times. Numerical experiments were conducted for the Asia-America-Europe service route. It was found that a liner shipping company could improve robustness of its schedule by adding more vessels. Brouer et al. studied a Vessel Schedule Recovery Problem (VSRP), taking into account disruptions that might occur in liner shipping. The problem was formulated as a mixed integer linear program. The following disruptive scenarios were modeled: a) vessel delays due to weather conditions, b) a port closure, c) a berth prioritization, when two vessels arrive simultaneously at the port and are scheduled at the same berth, and d) an expected port congestion. The following countermeasures were suggested to mitigate effects of the uncertainty: a) port omitting, b) increasing vessel speed, c) swap ports of call, and d) accept vessel delays. Generated problem instances were solved using CPLEX. It was found that the suggested methodology could yield up to 58% if the total cost savings.
Sheng, Chew, and Lee (10) proposed a dynamic \((s, S)\) policy for liner shipping refueling and sailing speed optimization problem, taking into account uncertainty in bunker prices and bunker consumption. According to the \((s, S)\) bunker refueling policy, the vessel gets the fuel refill up to \(S\), if the bunker inventory is below \(s\). The normal distribution was assigned to model the bunker consumption uncertainty, while the scenario analysis was used to model the bunker price uncertainty. The objective minimized the expected total cost. The SAA method was used to estimate the expected bunker consumption. A progressive hedge algorithm was developed to solve the problem. Numerical experiments indicated that the dynamic \((s, S)\) policy could yield substantial cost savings for liner shipping companies. Wang and Meng (11) studied a robust bunker management problem, where the real vessel speeds were assumed to deviate from the planned ones. The worst-case bunker consumption function was introduced to capture changes in the bunker consumption due to vessel speed deviations. The objective of a non-linear mixed integer mathematical model minimized the total transportation cost. The original model was linearized and solved using CPLEX. Numerical experiments were conducted for the Asia-Europe-Oceania network. Results indicated that increasing vessel speed variability increased the number of deployed vessel at each route. Increasing inventory cost caused increasing vessel sailing speeds and reduced the number of deployed vessels at each route. Selection of fuel refill ports was affected by discounts suggested.

Unlike the literature published to date on port handling time uncertainty in liner shipping scheduling, we propose in this report a deterministic formulation, where robustness is obtained by simultaneous minimization of the range and mean cost of the schedule (total vessel weekly operating cost, total bunker consumption cost, total port handling cost, and total late arrival penalty). We do not model uncertainty in vessel speed, as it is a parameter fully controlled by the liner shipping company (unlike handling time at each port of call). It is easier for a liner shipping company to adjust the vessel speed throughout the voyage and achieve the planned sailing time. On the other hand, actual handling times at the ports are (almost) out the control of the liner shipping company. The liner shipping company may request higher rates to compensate for delays (in real time), but this approach falls in the domain of reactive re-scheduling and is out of the scope of this report (i.e., pro-active scheduling). In particular, a bi-objective optimization problem is formulated, where the first objective minimizes the average total route service cost and the second objective minimizes the route service cost range. Total costs, associated with a schedule, heavily depend on the actual vessel handling times, and we define the average total route service cost as the arithmetic average of the possible maximum and possible minimum total route service costs. However, solely minimization of the average total route service cost may result in high variations in total route service costs, i.e. a cost-unstable schedule. Therefore, we also consider minimizing the route service cost range of a schedule, which is defined as the difference between the possible maximum and possible minimum total route service costs. It is worth noting that robustness/stability is an important concept studied in scheduling problems for various applications in uncertain environments. Robustness of a schedule may be defined as its insensitiveness to the uncertainties, and stability, performance range of a given schedule, can be used to measure robustness of a schedule (12). The cost range of a schedule, defined for the problem of interest in this report, is assumed to represent the stability of a schedule, i.e. it is used to measure the schedule’s robustness. We refer the reader to the book by Billaut et al. (12) for a detailed discussion on robustness in scheduling problems.
that arise in different practical applications. Next we present the problem description followed by the mathematical formulation.

PROBLEM DESCRIPTION
In this study we consider a liner shipping route with $I = \{1, \ldots, n\}$ ports of call (see Figure 1). Each port is assumed to be visited once and the sequence of visited ports (a.k.a. port rotation) is known. The former assumption does not limit generality of the suggested methodology and can be relaxed as needed, i.e., some ports can be visited more than once and represented by additional nodes. The latter decision is made at the strategic level (4). A vessel sails between two subsequent ports $i$ and $i + 1$ along leg $i$. The liner shipping company provides a weekly service at each port of call. The terminal operator at each port sets a specific arrival time window $TW = [t_{wi}^e - t_{wi}^l]$ – the earliest start at port $i$, $t_{wi}^l$ – the latest start at port $i$], during which a vessel should arrive at the port (can be up to 1-3 days depending on the port). Weekly demand (TEUs) at each port is known, while the quantity of containers transported by alliance partners is excluded from the total weekly demand, as this decision is usually made by the liner shipping company at the strategic level (4).

![FIGURE 1 Illustration of a Shipping Route.](image)

Vessel Service at Ports
Terminal operators have various contractual agreements with the liner shipping company, according to which each terminal operator offers a set of handling rates $S_i = \{1, \ldots, s_i\} \forall i \in I$ to the liner shipping company. If higher service rate is requested, the port handling time for a given vessel decreases, but port handling charges, imposed to the liner shipping company, increase. Due to uncertainty in marine container terminal operations the actual port handling time may deviate from the agreed one. To account for this uncertainty we assume that vessel handling time at each port is a variable, following a uniform distribution with known upper and lower bounds.

Vessel Arrivals
The following scenarios of vessel arrivals are considered in this study:
1. A vessel arrives within a pre-defined TW and no penalties are imposed to the liner shipping company (see Figure 2A).

2. A vessel arrives at port $i+1$ before the TW begins ($tw_{i+1}^e$) even when sailing at the lowest possible speed ($v_{min}$) (see Figure 2B). In this case we assume that the vessel waits at a dedicated area at port $i$ to ensure arrival within the TW at port $i+1$. The port waiting time ($wt_i$) can be calculated as: $wt_i = tw_{i+1}^e - \frac{l_i}{v_i} - t_i^d$ (Figure 2C), where $v_i$ is the sailing speed on leg $i$, $l_i$ is length of leg $i$, and $t_i^d$ is departure time from port $i$. Note that technically the vessel can also wait at port $i+1$, or split waiting times between ports $i$ and $i+1$. Future research may focus on evaluation of different decisions regarding the port waiting time.

3. A vessel arrives after the end of the TW ($tw_{i+1}^l$) (see Figure 2D), and monetary penalties are imposed to the liner shipping company (in USD/hr.), for service to start upon arrival (we assume that the liner shipping company under consideration can negotiate such an agreement). The penalty value is assumed to linearly increase with hours of late arrival ($lt_{i+1}$).

**FIGURE 2 Vessel Arrival Cases.**

**Bunker Consumption**

It is assumed that a vessel fleet for a given route is homogenous, which is common practice revealed in the literature (7, 8, 13), and the relationship between the bunker consumption and the vessel speed is as follows:
\( q(\overline{v}) = q(v^*) \times \left( \frac{\overline{v}}{v^*} \right)^\alpha = \gamma \times (\overline{v})^\alpha \)  

(1)

where:

- \( q(\overline{v}) \) – daily bunker consumption (tons of fuel/day);
- \( \overline{v} \) – average daily sailing speed (knots);
- \( q(v^*) \) – daily bunker consumption when sailing at the designed speed (tons of fuel/day);
- \( v^* \) – design sailing speed (knots);
- \( \alpha, \gamma \) – coefficients calibrated from the historical data;

Generally, an additional regression analysis is conducted to determine the values of \( \alpha \) and \( \gamma \) for each vessel in the fleet (13, 14). Due to lack of data, the most common values from the literature (13) are adopted in this report (i.e., \( a = 3 \) and \( \gamma = 0.012 \)). Once the liner shipping company decides on a sailing speed between consecutive ports, it is assumed to remain constant. Factors affecting the vessel speed during voyage (e.g., weather conditions, wind speed, height of waves, etc.) are not considered. The fuel consumption by auxiliary engines was included in the weekly vessel operating cost. Note that bunker consumption per nautical mile \( f(v_i) \) at leg \( i \) can be estimated as follows:

\[
f(v_i) = q(v_i) \times \left( \frac{t_i}{24} \right) \times \frac{1}{l_i} = \gamma \times (v_i)^\alpha \times \frac{l_i}{24} \times \frac{1}{v_i} = \frac{\gamma \times (v_i)^{a-1}}{24} \forall i \in I
\]

(2)

where:

- \( t_i \) – sailing time between ports \( i \) and \( i + 1 \) (hrs.)

**Decisions**

The problem, considered in this study, can be classified as a tactical level problem and will be referred to as the robust vessel schedule design problem. In this problem the liner shipping company determines the following:

1) The number of vessels assigned to the given route in order to provide weekly service at each port (decision on fleet size and mix is assumed to be made at the strategic level, 4)
2) The handling time (or handling rate) at each port, taking into account TW constraints and increasing charges for higher handling rates
3) The port waiting time to ensure feasibility of arrival at the next port of call
4) The sailing speed between consecutive ports, taking into account service TW constraints at each port, associated bunker consumption costs, and uncertainty in port handling times
5) The vessel late arrival fees.

A liner shipping company sets a maximum quantity of vessels that can be deployed at any given route \( (q \leq q^{\text{max}}) \) and sets limits on lower and upper vessel sailing speeds \( (v_i^{\text{min}} \leq v_i \leq v_i^{\text{max}} \forall i \in I) \). The minimum sailing speed \( v_i^{\text{min}} \) is selected to reduce wear of the vessel’s engine (15), while the maximum sailing speed \( v_i^{\text{max}} \) is defined by the capacity of the vessel’s engine (16).
All these decisions are interrelated. Selecting lower sailing speed reduces the bunker consumption, but may require deployment of more vessels at the given route to ensure that weekly service is met, which increases the total weekly operating cost (e.g., crew costs, maintenance, repairs, insurance, etc.). Various port handling rates further allow the liner shipping company to weigh different options between sailing and port handling times (e.g., faster handling rate reduces the service time at a given port, which may allow sailing at a lower speed to the next port of call). On the other hand, uncertainty in port handling times may require sail at higher speed (i.e., create a buffer time to account for uncertainty) to the next port of call or increase the number of deployed vessels at the given route. Furthermore, higher handling rates may lead to the vessel waiting, once service is completed (Figure 2C).

**MODEL FORMULATION**

**The Deterministic Vessel Scheduling Problem**

Assume that port handling times, associated with given handing rates, are known with certainty at each port of call. Then the deterministic mixed integer non-linear vessel schedule design problem **DVSDP** can be formulated as follows.

**Nomenclature**

**Sets**

\[ I = \{1, \ldots, n\} \]  
set of ports to be visited

\[ S_i = \{1, \ldots, s_i\} \]  
set of available handling rates\(^1\) at each port \(i\)

**Decision variables**

\[ v_i \forall i \in I \]  
vessel sailing speed at leg \(i\), connecting ports \((i)\) and \((i + 1)\)

\[ x_{is} \forall i \in I, s \in S_i \]  
=1 if handling rate \(s\) is selected at port \(i\) (=0 otherwise)

**Auxiliary variables**

\[ q \]  
number of vessels deployed at the given route

\[ t^a_i \forall i \in I \]  
arrival time at port \(i\) (hrs.)

\[ t^d_i \forall i \in I \]  
departure time from port \(i\) (hrs.)

\[ wt_i \forall i \in I \]  
waiting time of a vessel at port \(i\) (hrs.)

\[ t_i \forall i \in I \]  
vessel sailing time at leg \(i\), connecting ports \((i)\) and \((i + 1)\)

\[ f(v_i) \forall i \in I \]  
bunker consumption at leg \(i\) when sailing at speed \(v_i\) (tons of fuel/nmi)

\[ lt_i \forall i \in I \]  
vessel late arrival at port \(i\) (hrs.)

**Parameters**

\[ \beta \]  
unit bunker cost (USD/ton)

\[ c^{OC} \]  
vessel weekly operating cost (USD/week)

\[ c^{LT}_i \forall i \in I \]  
delayed arrival penalty at port \(i\) (USD/hr.)

\[ l_i \forall i \in I \]  
length of leg \(i\) (nmi)

\[ v^{min} \]  
minimum vessel sailing speed (knots)

\[ v^{max} \]  
maximum vessel sailing speed (knots)

---

\(^1\) Set of handling rates contains indexes of available handling rates (i.e., if a terminal operator at port \(i\) offers two handling rates 75 TEUs/hr. and 50 TEUs/hr., then \(S_i = \{1,2\}\))
maximum number of deployed vessels
the earliest start at port $i$ (hrs.)
the latest start at port $i$ (hrs.)
handling cost at port $i$ under handling rate $s$ (USD)
vessel handling time at port $i$ under handling rate $s$ (hrs.)

**DVSDP**

$$
\min Z(q, v, x, l, t) = [c^0 c q + \beta \sum_{i \in I} l_i f(v_i) + \sum_{i \in I, s \in S_i} tc_{is} x_{is} + \sum_{i \in I} c_i^T t_i] 
$$

Subject to:

$$
\sum_{s \in S_i} x_{is} = 1 \forall i \in I
$$

$$
t_i = \frac{l_i}{v_i} \forall i \in I
$$

$$
t_i^a \geq tw_i^a \forall i \in I
$$

$$
t_i^a + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i + t_i \geq tw_{i+1} \forall i < |I|
$$

$$
t_i^d + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i + t_i - 168q \geq tw_i \forall i = |I|
$$

$$
t_i^d = t_i^a + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i \forall i \in I
$$

$$
l_i \geq t_i^a - tw_i \forall i \in I
$$

$$
t_{i+1}^d = t_i^d + t_i \forall i < |I|
$$

$$
t_i^d = t_i^d + t_i - 168q \forall i = |I|
$$

$$
168q \geq \sum_{i \in I} t_i + \sum_{i \in I, s \in S_i} (p_{is} x_{is}) + \sum_{i \in I} wt_i
$$

$$
q \leq q_{\text{max}}
$$

$$
v_{\text{min}} \leq v_i \leq v_{\text{max}} \forall i \in I
$$

$$
x_{is} \in \{0, 1\} \forall i \in I, s \in S_i
$$

$$
q, q_{\text{max}} \in N
$$

$$
v_i, t_i^a, t_i^d, wt_i, t_i, f(v_i), l_i, \beta, c^0 c, c_i^T, l_i, v_{\text{min}}, v_{\text{max}}, p_{is}, tw_i^a, tw_i^d \in R^+ \forall i \in I, s \in S_i
$$

In **DVSDP** the liner shipping company minimizes the total route service cost (3), which includes 4 components: 1) total vessel weekly operating cost, 2) total bunker consumption cost, 3) total port handling cost, and 4) total late arrival penalty. Constraints set (4) indicate that only one handling rate can be selected at each port of call. Constraints set (5) calculate a vessel sailing time between ports $i$ and $i + 1$. Constraints set (6) ensure that a vessel cannot arrive at port $i$ before the agreed TW. Constraints sets (7) and (8) compute waiting time at port $i$, necessary to ensure feasibility of arriving at the next port of call. Constraints set (9) calculate a vessel departure time from port $i$. Constraints set (10) estimate hours of late arrival at port $i$. Constraints sets (11) and (12) compute a vessel arrival time at the next port of call. Constraints set (13) ensure weekly service frequency (168 denotes the total number of hours in a week). The right-hand-side of an equality estimates the total turnaroud time of a vessel at the given route (where the first component is the total sailing time, the second component is the total port handling time, and the third component is the total port waiting time). Constraints set (14) ensure that the number of vessels to be deployed at the given route should not exceed the number of
available vessels. Constraints set (15) define range of a sailing speed at leg $i$. Constraints (16) – (18) define ranges of parameters and variables.

The Robust Vessel Scheduling Problem

To solve $DVSDP$ port handling time $p_{is} \forall i \in I, s \in S_i$ should be given at each port of call. Nevertheless, as discussed in the third section of the paper, port handling times are subject to variability. In this study uncertain port handling time $p_{is}$ are modeled using a uniform distribution with known upper $p_{is}^u$ and lower bounds $p_{is}^l$: $p_{is}^l \leq p_{is} \leq p_{is}^u \forall i \in I, s \in S_i$ (17-19). Uncertain port handling times cause variability of the total route service cost. The liner shipping company aims to develop a robust vessel schedule by simultaneously minimizing the average route service cost (ARC) and route service cost range (RCR). Denote $X$ as $n \times s$ matrix that defines available handling rates at each port of call, when handling time associated with each service rate $p_{is}$ is not known with certainty. Denote $V$ as a vector with $n$ elements that defines sailing speed at each leg of a given route. Them the robust vessel schedule design problem with uncertain port handling times $RVSDP$ can be formulated as follows.

\[
RVSDP
\]

\[
ARC = \min [\max \{Z(q,v,x,lt)\} + \min \{Z(q,v,x,lt)\}] \\
RCR = \min [\max \{Z(q,v,x,lt)\} - \min \{Z(q,v,x,lt)\}] \\
\]

Subject to:
Constraints sets (4)-(18)

The objective function (19) minimizes the average route service cost, while the objective function (20) minimizes the route service cost range. Both objective functions contain two optimization problems (i.e., maximization and minimization of the total route service cost). To overcome this issue we reformulate $RVSDP$ as a bi-level bi-objective optimization problem $BRVSDP$ (19). Denote $[Q^{MAX},X^{MAX},LT^{MAX}]$ and $[Q^{MIN},X^{MIN},LT^{MIN}]$ as number of vessels, port handling rate, and hours of late vessel arrivals that maximize and minimize the total route service cost. $RVSDP$ can then be reformulated as follows:

\[
BRVSDP
\]

Upper Level (UL):
\[
ARC = \min_{v,x} \left[ \frac{1}{2} \{Z(Q^{MAX},V,X^{MAX},LT^{MAX}) + Z(Q^{MIN},V,X^{MIN},LT^{MIN})\} \right] \\
RCR = \min_{v,x} \left[ \{Z(Q^{MAX},V,X^{MAX},LT^{MAX}) - Z(Q^{MIN},V,X^{MIN},LT^{MIN})\} \right]
\]

Subject to:
Constraints sets (4)-(18)

Lower Level (LL):
\[ \begin{align*}
[Q_{\text{MAX}}, X_{\text{MAX}}, L_{\text{MAX}}] &= \text{argmax}\{Z(q, v, x, lt)\} \\
[Q_{\text{MIN}}, X_{\text{MIN}}, L_{\text{MIN}}] &= \text{argmin}\{Z(q, v, x, lt)\}
\end{align*} \] (23)

Subject to:
Constraints sets (4)-(18)

SOLUTION ALGORITHM
Bi-level optimization problems are (in the majority of cases) non-convex and difficult to solve using the exact optimization methods (18, 19). A bi-objective evolutionary algorithm (BEA) was developed to solve BRVSDP, as EAs are widely used in different fields to solve problems of a high complexity (20). The main BEA steps are outlined in Procedure 1. In step 1, the chromosomes (i.e., population) are initialized by solving DVSDP with random (but deterministic) handling times at each port PopSize times, where PopSize denotes the population size. In step 2, LL problems are solved and UL objective functions are evaluated, while in step 3 the Pareto Front is determined. While the stopping criterion is not met, BEA selects parents, mutates them, and goes to step 2. Next we describe each component of the developed BEA in detail.

Procedure 1. Bi-Objective Evolutionary Algorithm

Step 0: Set gen=1
Step 1: Initialize chromosomes
Step 2: Solve LL problems and evaluate UL objective functions
Step 3: Determine Pareto Front
Step 4: If stopping criterion is met \( \rightarrow \) End, else go to step 5
Step 5: Set gen=gen+1
Step 6: Select parents and mutate
Step 7: Go to step 2

Chromosome Representation
A real value chromosome was used in the developed BEA to represent a solution (i.e., vessel sailing speeds at each leg of the route). An example a chromosome is presented in Figure 3A for a portion of liner shipping route with 8 ports of call. The chromosome includes the following information: a) leg, and b) vessel sailing speed. For example, the sailing speed at the 3\textsuperscript{rd} leg, connecting 3\textsuperscript{rd} and 4\textsuperscript{th} ports, is 23.0 knots.
Chromosomes and Population Initialization
Chromosomes were initialized by changing handling time at each port of call for each handling rate $p_{is} = U[p_{is}^l; p_{is}^u] \forall i \in I, s \in S_i$, where $U$ denotes uniformly distributed pseudorandom numbers. Different population sizes $PopSize$ (i.e., number of chromosomes in the population) will be evaluated during the numerical experiments. The population size does not change throughout the BEA evolution. Vessel sailing speeds were initialized by solving $DVSDP$ for each individual in the population and then used as an input for $LL$.

$DVSDP$ can be linearized by replacing sailing speed $v_i$ with its reciprocal $y_i$ and the non-linear bunker consumption function $G(y)$ with its secant approximation $\bar{G}_m(y)$, where $m$ is the number of segments (see Figure 3B). Let $K = \{1, 2, ..., m\}$ be the set of linear segments of the piecewise function $G_m(y)$. Let $b_{ik} = 1$ if segment $k$ is selected for approximation of the bunker consumption function at leg $i$ ($=0$ otherwise). Denote as $st_k, ed_k, k \in K$ the speed reciprocal values at the start and end (respectively) of linear segment $k$; $SL_k, IN_k, k \in K$ the slope and an intercept of linear segment $k$ (obtained from a piecewise linear regression analysis); and $M_1, M_2$ as sufficiently large positive numbers. Then, $DVSDP$ can be reformulated as a linear problem ($DVSDPL$) as follows.
**DVSDPL**

\[
\min Z(q, y, x, l) = [c^{OC}q + \beta \sum_{i} l_i \sum_{k} G_k(y_i)] + \sum_{i} \sum_{s} tc_{is}x_{is} + \sum_{i} c^l t_i
\]  

(25)

**Subject to:**

Constraints sets (4), (6)-(14), (16)-(18)

\[
\sum_{k} b_{ik} = 1 \forall i \in I
\]

(26)

\[
st_k \times b_{ik} \leq y_i \forall i \in I, k \in K
\]

(27)

\[
ed_k + M_1 \times (1 - b_{ik}) \geq y_i \forall i \in I, k \in K
\]

(28)

\[
G_k(y_i) \geq SL_k \times y_i + IN_k - M_2 \times (1 - b_{ik}) \forall i \in I, k \in K
\]

(29)

\[
t_i = l_i \times y_i \forall i \in I
\]

(30)

\[
1/v^{max} \leq y_i \leq 1/v^{min} \forall i \in I
\]

(31)

In **DVSDPL** objective (25) minimizes the total route service cost. Constraints set (26) ensure that only one segment \(k\) will be selected for approximation of the bunker consumption function at leg \(i\). Constraints sets (27) and (28) define range of vessel sailing speed reciprocal values, when segment \(k\) is selected for approximation of the bunker consumption function at leg \(i\). Constraints set (29) estimate the approximated bunker consumption at leg \(i\). Constraints set (30) calculate a vessel sailing time between ports \(i\) and \(i + 1\). Constraints set (31) show that a reciprocal of vessel sailing speed should be within specific limits. Strict bounds for \(M_1, M_2\) can be defined as follows: \(M_1 = \frac{1}{v^{min}}\), \(M_2 = SL_1 \times \frac{1}{v^{max}} + IN_1\). The number of segments in the secant approximation was set to 4 (see \(G_4(y)\) in Figure 3B), as increasing number of segments will increase **DVSDPL** computational time without significant change in the objective function value.

**Fitness Function**

For **EAs** the fitness function is usually associated with the objective function (20). In the proposed **BEA** the fitness function value was set equal to the objective function value for both ARC and RCR without applying any scaling mechanisms. In order to estimate the objective function values at **UL** of **BRVSDP**, it is necessary to solve the two optimization problems of the **LL**. Denote as \(M_3\) as a sufficiently large positive number. Then, both **LL** problems can be formulated as follows.

**LL**

The objective (32) maximizes/minimizes the total route service cost.

\[
\max/\min_{x} x\{Z(q, v, x, l)\}
\]

(32)

**Subject to:**

Constraints sets (4)-(6), (10)-(12), (14), (16-18)

\[
p^l_{is}x_{is} \leq p_{is} \forall i \in I, s \in S_i
\]

(33)

\[
p_{ls} \leq p^u_{is}x_{is} \forall i \in I, s \in S_i
\]

(34)

\[
t_{i}^l + \sum_{s} (p_{is}) + wt_{i} + t_i \geq tw_{i+1} \forall i < |I|
\]

(35)

\[
t_{i}^l + \sum_{s} (p_{is}) + wt_{i} + t_i - 168q \geq tw_{1} \forall i = |I|
\]

(36)
In **LL** constraints sets (33) and (34) estimate handling time at port $i$ under selected handling rate $s_i$. Constraints sets (35) and (36) compute waiting time at port $i$, necessary to ensure feasibility of arriving at the next port of call. Constraints set (37) calculate a vessel departure time from port $i$. Constraints set (38) ensure weekly service frequency (168 denotes the total number of hours in a week). Constraints set (39) ensure that the hours of vessel late arrival at port $i$ are less than its upper bound. Value of $M_3$ may vary from port to port and can be assigned based on historical data with vessel late arrivals at ports of the given route. Introduction of constraint set (39) is crucial to ensure that **LL** will not become unbounded.

**Pareto Front Selection**
Since **BRFDP** is bi-objective, it is necessary to determine the Pareto Front (21). The Pareto Front refers to a set of non-dominant solutions, which meet both objectives (ARC and RCR), and there is no other solution that can improve at least one of the objectives without worsening the other objective (21). The Pareto Front selection is outlined in Procedure 2.

**Parent Selection**
The parent selection at a given **BEA** generation is an important part of its design (20). In the developed **BEA** the parents were randomly selected from Pareto Fronts, obtained in the last GenPF = 5 generations.

**BEA Operations**
A custom **BEA** operator was developed to produce the offspring. The **BEA** operator changes vessel sailing speed as follows: $v_i = U[v_{\min}; v_{\max}]$. An example of the **BEA** operation at legs 2 and 6 is presented in Figure 3C. Sailing speed at the 2\textsuperscript{nd} leg changes from 10.3 to 12.7 knots, while sailing speed at the 6\textsuperscript{th} leg changes from 19.8 to 15.8 knots. The number of legs to be mutated during the **BEA** operation is determined based on the mutation rate MutRate.

**Procedure 2. Pareto Front Selection**

```
\textbf{ParetoFront}(Offspring\_gen, Fit\_gen)
\textbf{in:} Offspring\_gen - offspring produced at the given generation; Fit\_gen - offspring fitness values,
\textbf{Fit\_gen} = [ARC\_gen, RCR\_gen]
\textbf{out:} PF - Pareto Front
1: |PF| ← |Offspring\_gen| \hspace{1cm} ← Initialization
2: i ← 1
3: for $i \in Offspring\_gen$ do
4: \hspace{1cm} j ← i + 1
5: \hspace{1cm} for all $j \in Offspring\_gen$ do
6: \hspace{2cm} if $ARC_i < ARC_j$ and $RCR_i < RCR_j$ \hspace{1cm} ← Check if solution \{j\} is dominated by solution \{i\}
```

\[ t_i^d = t_i^a + \sum_{s \in S_i} (p_{is}) + wt_i \forall i \in I \]  
\[ 168q \geq \sum_{i \in I} t_i + \sum_{i \in I} \sum_{s \in S_i} (p_{is}) + \sum_{i \in I} wt_i \]  
\[ lt_i \leq M_3 \forall i \in I \]
7: \[ PF \leftarrow PF - \{ j \} \]
8: \[\text{else if } ARC_i > ARC_j \text{ and } RCR_i > RCR_j \rhd \text{Check if solution } \{ i \} \text{ is dominated by solution } \{ j \} \]
9: \[ PF \leftarrow PF - \{ i \} \]
10: \[\text{end if} \]
11: \[ j \leftarrow j + 1 \]
12: \[\text{end for} \]
13: \[ i \leftarrow i + 1 \]
14: \[\text{end for} \]
15: \[\text{return } PF \]

**Stopping Criterion**

In this report the algorithm was terminated after a pre-specified number of generations (\textit{LimitGen} of 200 generations).

**NUMERICAL EXPERIMENTS**

This section presents numerical experiments to showcase efficiency of the proposed methodology for a real-life liner shipping route.

**Input Data Description**

This study considers the New North Europe Med Oceania route (see Figure 4\(^2\)), served by CMA CGM liner shipping company. This route connects Europe, Australia, Singapore, Malaysia, India, and Sri Lanka. The port rotation for New North Europe Med Oceania route includes 18 ports of call (distance to the next port of call in nautical miles is presented in in parenthesis, estimated using world seaports catalogue\(^3\)):

1. Tilbury, GB (531) \rightharpoonup 2. Hamburg, DE (341) \rightharpoonup 3. Rotterdam, NL (335) \rightharpoonup 4. Le Havre, FR (2,365) \rightharpoonup 5. Genoa, IT (1,766) \rightharpoonup 6. Damietta, EG (7,715) \rightharpoonup 7. Fremantle, AU (1,720) \rightharpoonup 8. Melbourne, AU (628) \rightharpoonup 9. Sydney, AU (1,048) \rightharpoonup 10. Adelaide, AU (4,054) \rightharpoonup 11. Singapore, SG (230) \rightharpoonup 12. Port Kelang, MY (1,661) \rightharpoonup 13. Chennai, IN (401) \rightharpoonup 14. Colombo, LK (353) \rightharpoonup 15. Cochin, IN (3,558) \rightharpoonup 16. Damietta, EG (1,103) \rightharpoonup 17. Malta, MT (711) \rightharpoonup 18. Genoa, IT (2,388) \rightharpoonup 1. Tilbury, GB

\(^2\) http://www.cma-cgm.com/products-services/line-services/flyer/FAL (accessed on 06/01/2015)
\(^3\) http://ports.com/sea-route
The required numerical data were generated based on the available liner shipping literature (22-26) and are presented in Table 1. The latest start at each port of call was set based on the latest start at preceding port, leg length between subsequent ports, and vessel sailing speed bounds: $\text{tw}_{i+1}^l = \text{tw}_i^l + \frac{l_i}{U_{\text{v}_{\text{min}},\text{v}_{\text{max}}}} \forall i \in I$. The duration of a TW ($\text{tw}_i^l - \text{tw}_i^e$) was assigned as $U[24; 72]$ hrs. \(22\). Weekly demand $NC_i$ (in TEUs) at large ports was generated as $U[500; 2,000]$ TEUs. Note that term “large port” was applied to those ports of call, if they were included in the list of top 20 world container ports based on their throughput \(23\). Weekly demand at smaller ports was assigned as $U[200; 1,000]$ TEUs. Large ports were able to offer the following handling productivities: \(125; 100; 75; 50\) TEUs/hr. Smaller ports could provide either \(100; 75; 60; 50\) or \(75; 70; 60; 50\) TEUs/hr. The latter assumption can be explained by the fact that terminal operators at large ports usually have more vessel handling equipment available and can offer more handling rate options to the liner shipping company. Furthermore, higher amounts of TEU handled can increase productivity. Upper and lower bounds for port handling times were assigned as follows: $p_{is}^{u} = \gamma_{is}^{u} \left(\frac{NC_i}{r_{is}}\right); p_{is}^{l} = \gamma_{is}^{l} \left(\frac{NC_i}{r_{is}}\right) \forall i \in I, s \in S_i$, where $\gamma_{is}^{u}$ and $\gamma_{is}^{l}$ – coefficients, showing change in port handling time due to uncertainty, and $r_{is}$ – handling productivity at port $i$ under handling rate $s$. Coefficients $\gamma_{is}^{u}$ and $\gamma_{is}^{l}$ were assumed to be \(1.25; 1.20; 1.15; 1.10\) and \(0.90; 0.92; 0.94; 0.96\) respectively. A realization of the uncertain port handling time was computed as: $p_{is} = U[p_{is}^{l}; p_{is}^{u}] \forall i \in I, s \in S_i$.

<table>
<thead>
<tr>
<th>Table 1 Numerical Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunker consumption coefficients $\alpha, \gamma$</td>
<td>$\alpha = 3, \gamma = 0.012$</td>
</tr>
<tr>
<td>Unit bunker cost $\beta$ (USD/ton)</td>
<td>750</td>
</tr>
<tr>
<td>Vessel weekly operating cost $c^{OC}$ (USD/week)</td>
<td>300,000</td>
</tr>
<tr>
<td>Delayed arrival penalty $c^{LT}_i$ (USD/hr.)</td>
<td>$U[5,000; 10,000]$</td>
</tr>
</tbody>
</table>
### Table

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upped bound on late arrival $M_4$ (hrs.)</td>
<td>U[24; 72]</td>
</tr>
<tr>
<td>Minimum vessel sailing speed $v^{\text{min}}$ (knots)</td>
<td>14</td>
</tr>
<tr>
<td>Maximum vessel sailing speed $v^{\text{max}}$ (knots)</td>
<td>24</td>
</tr>
<tr>
<td>Maximum number of deployed vessels $q^{\text{max}}$</td>
<td>15</td>
</tr>
</tbody>
</table>

The handling cost per TEU $s_{ci}$ at each port $i$ under handling rate $s$ was computed as:

$$s_{ci} = asc \pm U[0; 50] \forall i \in I, s \in S_i,$$

where $asc$ is the average container handling cost. Then the total handling cost was computed as:

$$t_{ci} = s_{ci} \cdot NC_i \forall i \in I, s \in S_i.$$

Based on the available literature (24, 25) and assuming a mix of vessel operations that include mooring, loading and discharge of containers, type of container (empty, loaded, size, reefer), re-stowing (on-board the vessel or via quay), the average container handling cost was set equal to [700; 625; 550; 475] USD/TEU for four available handling rates respectively. It was assumed that each terminal operator perceives handling cost differently (i.e., service charge for the same handling rate varies from port to port), which is accounted for by the second (and random) term of the $s_{ci}$ formula. Note that if the handling time, associated with the selected handling rate, changes due to uncertainty, the liner shipping company is still expected to pay the negotiated handling cost $t_{ci}$.

All numerical experiments were conducted on a Dell T1500 Intel(T) Core i5 Processor with 1.96 GB of RAM. A static secant approximation for the bunker consumption function was developed using MATLAB 2014a. Lower level problems $LL$ were solved using CPLEX of General Algebraic Modeling System (GAMS) for each individual in the population at each generation of $BEA$.

### Proposed Methodology Evaluation

A total of five problem instances were generated using the numerical data from the previous subsection by changing service TWs at ports and TW duration. The developed $BEA$ with $\text{PopSize} = 20$ and $\text{MutRate} = 1$ was executed for each one of those instances, and the Pareto Fronts (PFs) obtained at termination are presented in Figure 5 (left). Preliminary experiments indicated that decreasing $\text{PopSize}$ and increasing $\text{MutRate}$ worsened the quality of the PF solutions and reduced the PF size. The $BEA$ computational time averaged on 30.6 min over 5 problem instances (see Table 2A). Relatively large computational time can be explained by the fact that $BRVSDP$ lower level problems were solved using CPLEX for each individual in the population at each generation of $BEA$ (a total of $2 \cdot \text{LimitGen} \cdot \text{PopSize} = 8,000$ lower level problems). Application of the optimization solver guarantees optimality for $BRVSDP$ lower level problems, but affects computational time of the solution algorithm.
TABLE 2 Numerical Experiments Results

<table>
<thead>
<tr>
<th>a) BEA Results</th>
<th>Instance 1</th>
<th>Instance 2</th>
<th>Instance 3</th>
<th>Instance 4</th>
<th>Instance 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generations</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>PF size</td>
<td>43</td>
<td>23</td>
<td>33</td>
<td>30</td>
<td>14</td>
</tr>
<tr>
<td>CPU time, sec</td>
<td>1939</td>
<td>1775</td>
<td>1844</td>
<td>1802</td>
<td>1813</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b) Comparative Simulation Results</th>
<th>Instance 1</th>
<th>Instance 2</th>
<th>Instance 3</th>
</tr>
</thead>
</table>

To evaluate the proposed methodology we performed a comparative simulation analysis for the schedules of the PF and deterministic vessel schedule for each one of the five instances. The latter schedules were obtained by solving DVSDP with $p_{is} = \frac{p_{is}^l + p_{is}^u}{2}$ $\forall i \in I, s \in S_i$ (i.e., mean DVSDP or MDVSDP). The simulation analysis consisted in the following steps: 1) Generate 500 scenarios by changing realization of vessel handling times $p_{is} = U[p_{is}^l; p_{is}^u] \forall i \in I, s \in S_i$; 2) For each scenario update the vessel handling time under handling rates, selected by the PF-based and deterministic vessel schedules; 3) For each scenario recalculate the total route service cost for the PF-based and deterministic vessel schedules; 4) Determine the average route service cost and the route service cost range over 500 scenarios for the PF-based and deterministic vessel schedules (i.e., obtain simulated PFs - SPF and simulated MDVSDP). Results from the comparative simulation analysis are presented in Table 2B and Figure 5 (right). We observe that on average over all instances and simulation runs PF-based vessel schedules have 12.0% higher route service cost as compared to deterministic vessel schedules, but 56.8% lower route service cost range. Furthermore, certain PF-based schedules did not change their route service costs during simulation runs after changing $p_{is}$ $\forall i \in I, s \in S_i$ (i.e., zero route service cost range). The latter can be explained by the fact that those PF-based schedules had additional waiting times at some of the ports, and changing handling times only reduced waiting times without affecting other DVSDP variables and objective function. Thus, the proposed methodology outperformed the deterministic approach in terms of the route service cost range (i.e., vessel schedule reliability) and can be used by liner shipping companies in design of robust vessel schedules.

### CONCLUSIONS AND FUTURE RESEARCH AVENUES

Increasing port congestion and unexpected disruptive events at marine container terminals reduce reliability of liner shipping services. This report proposes a novel approach for modeling uncertainty in port handling times and presents a mixed integer non-linear mathematical model, directed to minimize the average route service cost and the route service cost range. Due to complexity of the original formulation, the model was reformulated as a bi-level problem. A bi-objective evolutionary algorithm was developed to solve the problem. Numerical experiments were performed for the New North Europe Med Oceania route, served by CMA CGM liner
shipping company. Results demonstrated that the suggested methodology provided vessel schedules with 12.0% higher route service cost as compared to the deterministic approach, but 56.8% lower route service cost range. Future research may focus on: 1) development of efficient heuristics to solve the lower level problems, 2) design of a more advanced parent selection operator, 3) testing the proposed methodology for different liner shipping routes, and 4) modeling uncertain handling times using different probability distributions.

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REFERENCES