Answer any three of the following six questions.  
You should state clearly any general results you use.

1. Classify all groups of order $45 = 3^2 \cdot 5$ stating clearly any results that you use.

2. Let $A$ be an abelian group and let $D = \{(a, a) : a \in A\}$ be the set of ‘diagonal elements’ in $A \times A$. Show that $D$ is a subgroup of $A \times A$ and that $(A \times A)/D \cong A$.

3. Let $f : G \to G$ be given by $f(x) = x^2$. Show that $f$ is a homomorphism if and only if $G$ is abelian.

4. Let $R$ be a commutative ring with 1.
   
   (a) Show that every maximal ideal is prime.
   
   (b) Show that if $R$ is a PID then every non-zero prime ideal is maximal.

5. A Boolean ring is a ring in which $x^2 = x$ for all $x$. Let $R$ be a commutative Boolean ring (with 1).
   
   (a) Show that $2x = 0$ for all $x \in R$.
   
   (b) Show that every prime ideal of $R$ is maximal.

6. Find the gcd of the elements $5 + 7i$ and $3 - 5i$ in $\mathbb{Z}[i]$. 
