Answer any three of the following six problems.

1. Let $G$ be a group of order $135 = 3^3 \cdot 5$. Show that $G$ is not simple.

2. Let $G$ be a group and $A, B$ two subgroups of $G$. Let $|A|$ denote the order of $A$. Suppose that $|A| > \sqrt{|G|}$ and $|B| > \sqrt{|G|}$. Prove that $A \cap B \neq \{e\}$.

3. Let $G$ be a finite group, $H$ a subgroup of $G$ and $N$ a normal subgroup of $G$. Suppose that $|H|$ and $|G : N|$ are relatively prime. Show that $H$ is a subgroup of $N$.

4. Let $R$ be a commutative ring with identity. Show that if $M$ is a maximal ideal of $R$, then the quotient ring $R/M$ is a field.

5. State the definition of Euclidean Domain. Prove that if $(a) = (b)$ for some elements $a, b$ in an integral domain $R$, then $a = ub$ for some unit $u$ of $R$.

6. Show that every nonzero prime ideal in a Principal Ideal Domain is a maximal ideal.