Master Exam-2006 Spring
Algebra I

Answer any three of the following six problems.

1. Let G be a group of order $135 = 3^3 \cdot 5$. Show that G is not simple.

2. Let G be a group and A, B two subgroups of G. Let $|A|$ denote the order of A. Suppose that $|A| > \sqrt{|G|}$ and $|B| > \sqrt{|G|}$. Prove that $A \cap B \neq \{e\}$.

3. Let G be a finite group, H a subgroup of G and N a normal subgroup of G. Suppose that $|H|$ and $|G : N|$ are relatively prime. Show that H is a subgroup of N.

4. Let R be a commutative ring with identity. Show that if M is a maximal ideal of R, then the quotient ring R/M is a field.

5. State the definition of Euclidean Domain. Prove that if $(a) = (b)$ for some elements a, b in an integral domain R, then $a = ub$ for some unit u of R.

6. Show that every nonzero prime ideal in a Principal Ideal Domain is a maximal ideal.