Do three of following problems.

1. Let \( f(x) \) be an integrable function on \( \mathbb{R} \) and for each \( n \), let \( h_n(t) = \frac{t^{2n}}{2 + t^{2n}} \). Show that the limit \( \int_{n \to \infty} f(t)h_n(t)dt \) exists and find its limit.

2. The outer measure on \( \mathbb{R} \) is defined by

\[
m^*(A) = \inf \left\{ \sum_{n=1}^{\infty} \ell(I_n) : I_n \text{ are open intervals such that } A \subseteq \bigcup I_n \right\}.
\]

(\( \ell(I) \) denotes the length of the interval \( I \)) Show that for any countable subsets \( E_n \) of \( \mathbb{R} \), \( m^*(\cup E_n) \leq \sum_n m^*(E_n) \). (countable subadditivity).

3. Let \( \{f_n\} \) be a sequence of measurable function that converges to \( f \) in measure. Show there is a subsequence of \( \{f_n\} \) that converges to \( f \) a.e.

4. State the Minkowski and Hölder inequalities.

5. Let \( \{f_n\} \) be a sequence of nonnegative measurable functions on \( \mathbb{R} \) such that \( f_n \) converges to \( f \) a.e. Suppose that \( f \) is integrable and \( \lim_{n \to \infty} \int f_n(t)dt = \int f(t)dt \). Show that for any measurable subset \( E \) of \( \mathbb{R} \),

\[
\lim_{n \to \infty} \int_E f_n(t)dt = \int_E f(t)dt.
\]