Do six of the following eight problems. But you need to do at least one of each of the following area Real Analysis (1-3), Algebra (4-5), and Topology (6-7).

1. Use the $\delta$-$\epsilon$ proof to show that the product of two continuous functions is continuous.

2. (a) Give an example of a sequence $(f_n)$ of continuous functions that converges to $f$ pointwise that is not continuous.
   
   (b) Show that if $(f_n)$ is a sequence of continuous functions that converges to $f$ uniformly. Then $f$ is continuous.

3. Let $f$ be a function from $[a, b]$ to $\mathbb{R}$ such that $f$ is differentiable at $c \in (a, b)$. Show that $f$ is continuous at $c$.

4. Let $M, N$ be two normal subgroups of $G$. Suppose that $M \cap N = \{e\}$. Show that $mn = nm$ for all $n \in N$ and $m \in M$.

5. Let $F \subseteq K \subseteq L$. Suppose that the dimension of $L$ over $K$ is $m(< \infty)$ and the dimension of $K$ over $F$ is $n(< \infty)$. What is the dimension of $L$ over $F$? Prove your argument.

6. Let $K$ be a compact set and $f$ a continuous from $K$ to $\mathbb{R}$. Show that there is $x \in K$ such that
   
   $$f(x) = \sup\{f(y) : y \in K\}.$$

7. Show that every metric space is first countable. Give an example of metric space that is not second countable.

8. Let $T$ be a linear transformation from a vector space $X$ to another vector space $Y$. Show that ker($T$) is a subspace of $X$. 