Review of Linear Algebra

$$
\begin{bmatrix}
1 & 0 & \cdots & 0 & 0 & c_{1r+1} & \cdots & c_{1n} \\
0 & 1 & \cdots & 0 & 0 & c_{2r+1} & \cdots & c_{2n} \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & \cdots & 1 & 0 & c_{r-1r+1} & \cdots & c_{r-1n} \\
0 & 0 & \cdots & & 1 & c_{rr+1} & \cdots & c_{rn} \\
0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & & & \vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix}
$$

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Matrix

- Definition
- Elements of a matrix (a_{ij})
- Vector (in bold or with an overline)
  - Row vector
  - Column Vector
- Scalar multiplication of two vectors
- Zero matrix ($\mathbf{0}$)
- Identity matrix ($\mathbf{I_m}$)

Matrix Operations

- Scalar multiple (***cA***)
- Addition of two matrices (***A+B***)
- Transpose of a matrix ($\boldsymbol{A}^{\boldsymbol{T}}$)
- Matrix multiplication (***AB***)
  - Associative ((***AB***)***C = A***(***BC***))
  - Distributive ((***A+B***)***C = AB+AC***))
  - Not commutative (***AB*** ≠ ***BA***)

Gauss-Jordan Method

- A system of linear equations has either
  - No solution, or
  - A unique solution, or
  - An infinite number of solutions
- This method specifies three basic operations on a matrix that can simplify solving a system of linear equations

Linear Independence

- Given a set of *m* row (or column vectors), they are all linearly independent if the only linear combination that equals ***0*** is their trivial combination,
- Otherwise, they are linearly dependent.

Rank

- Rank of a set of vectors is
  - the number of vectors in the largest linearly independent subset in the set.
- A matrix is a set of row vectors
- How to determine the rank of a matrix?
  - Apply the Gauss Jordan Method
  - The number of nonzero rows is its rank
- How to determine the rank of a set of vectors?