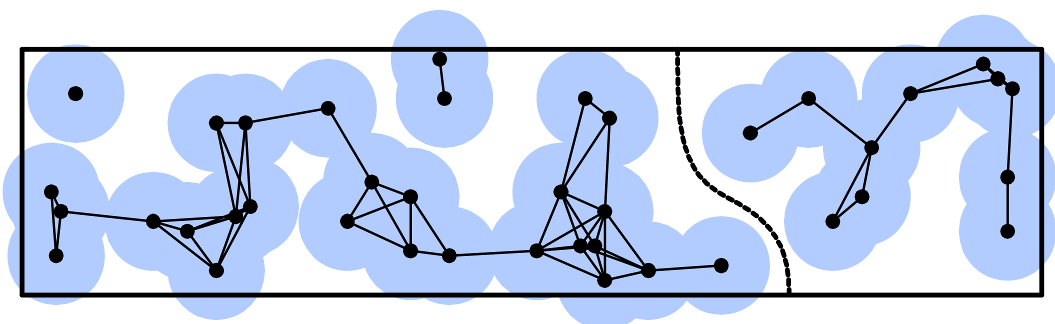
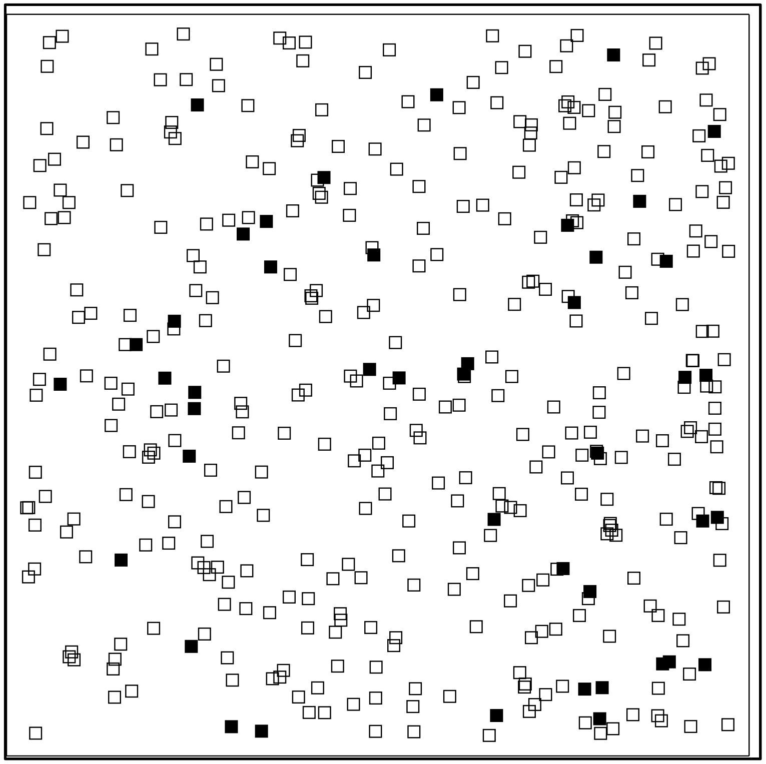
THE UNIVERSITY OF MEMPHIS
Computer Science

Coverage and Connectivity in Wireless Networks

Journey from Percolation to Reliable Density Estimates



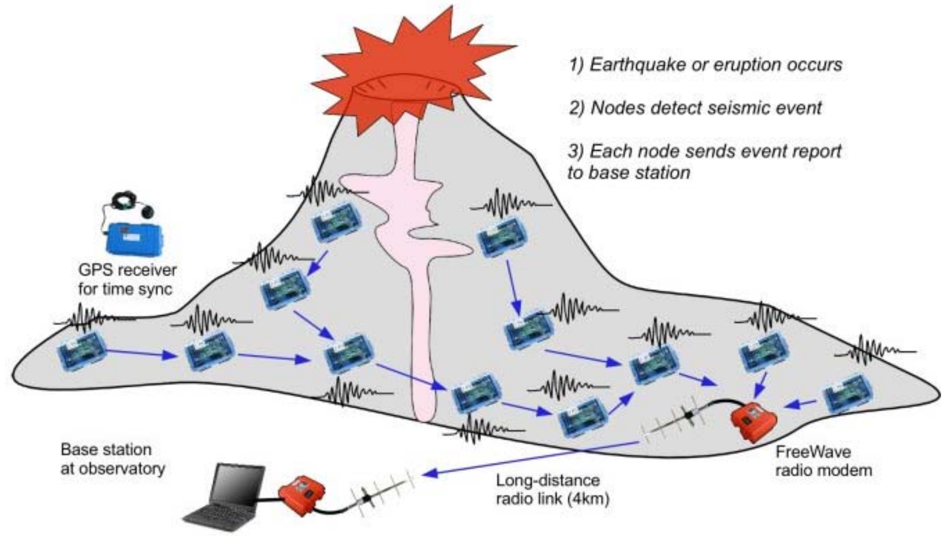


Santosh Kumar
University of Memphis

What is a Wireless Sensor?

- Extreme Scaling Mote (XSM)
  - CPU: 7.3827 MHz
  - Program Memory: 128 KB
  - RAM: 4 KB
  - Persistent storage: 4 MB
  - Radio: 30 m, 19.2 kbps
  - Sensors (range: 7 m – 30 m)
    - Infrared (to detect motion)
    - Acoustic (to detect sound)
    - Magnetic (to detect metal), etc.
  - Runs on a pair of AA batteries

Applications



Current Projects in Our Lab

- AutoSense (Sponsor: NIH)
- AutoWitness (Sponsor: NSF)
- Barrier Coverage (Sponsor: NSF)

AutoSense

- A body area sensor network to measure alcohol and stress exposure from the field
- High potential
  - Can study interaction between stress and addiction
  - Can study other addictive substances
  - Can study health issues remotely
- Trans-disciplinary team
  - Disciplines: CS, ECE, Behav. Sc., Biochem, Mat. Sc.
  - Organizations: UoM, OSU, UMN, SpectRx Inc.

AutoWitness

- Detect and track burglars
- Make a difference to the local society
- A foundational approach
  - Optimal deployment
  - System based on optimal or approximation algorithms
- Collaboration between CS and Mathematics

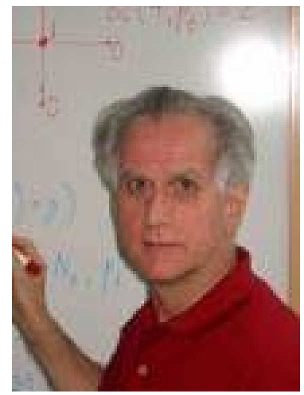
Barrier Coverage

- Mostly theoretical
  - Goal is to put intrusion detection on firm foundations
- Proposed the barrier coverage concept
  - MobiCom 2005
- Developed optimal solutions for several fundamental problems
  - Sleep wakeup (Broadnets 2007)
  - Localized barrier coverage (MobiCom 2007)
  - Reliable density estimate (MobiCom 2007)
- Investigating quality of coverage, and effects of directional and mobile sensors

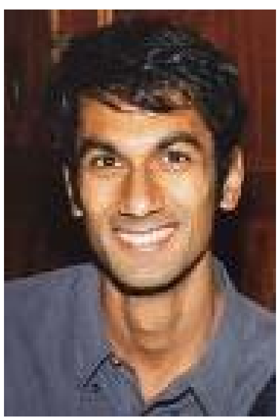
Collaborators

- Rest of the talk is based on our MobiCom 2007 paper with

Dr. Paul Balister



Dr. Béla Bollobás

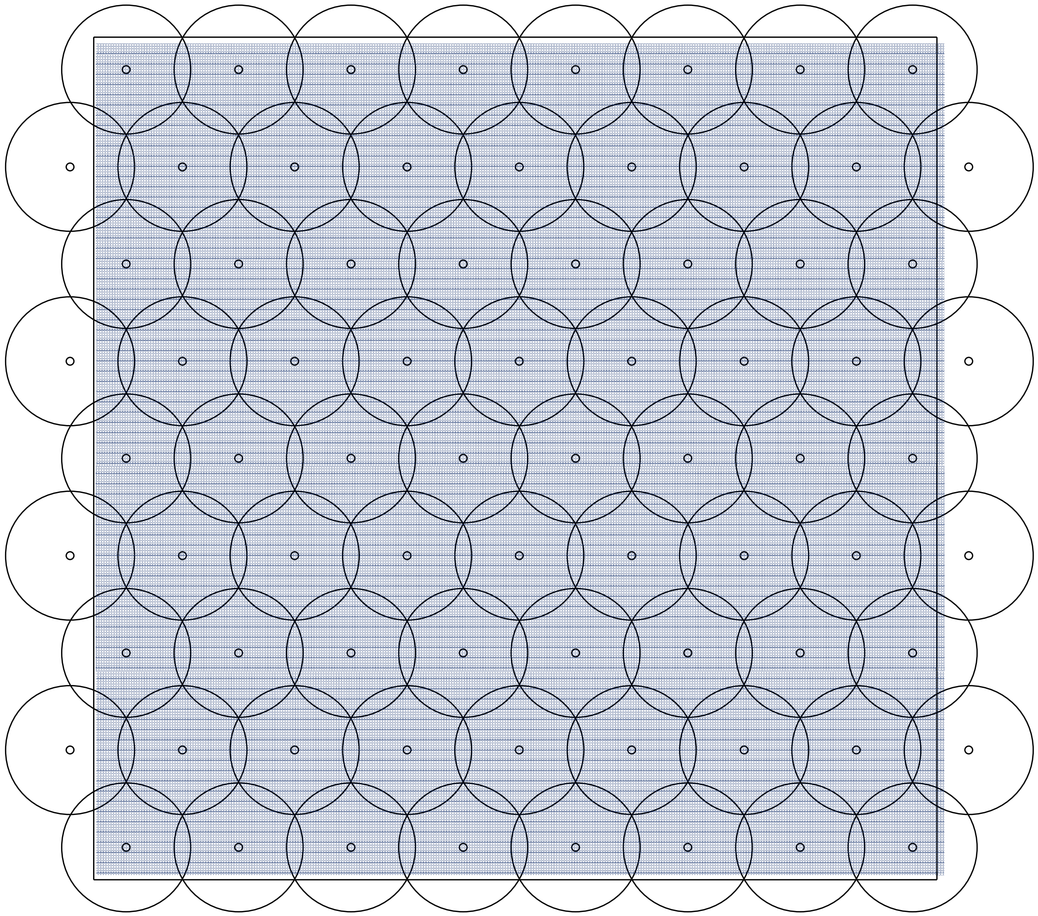


Dr. Amites Sarkar

Outline

- Defining coverage and connectivity
- Model
- Asymptotic probabilistic condition and its limitations
- Reliable density estimates – introduction and its application to full connectivity
- Deriving reliable density estimate for barrier coverage in thin strips

Full Coverage

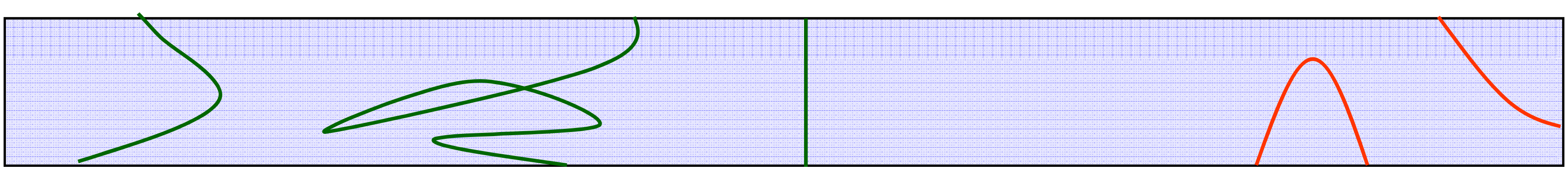


Crossing Paths

- A crossing path is a path that crosses the complete width of the belt region.

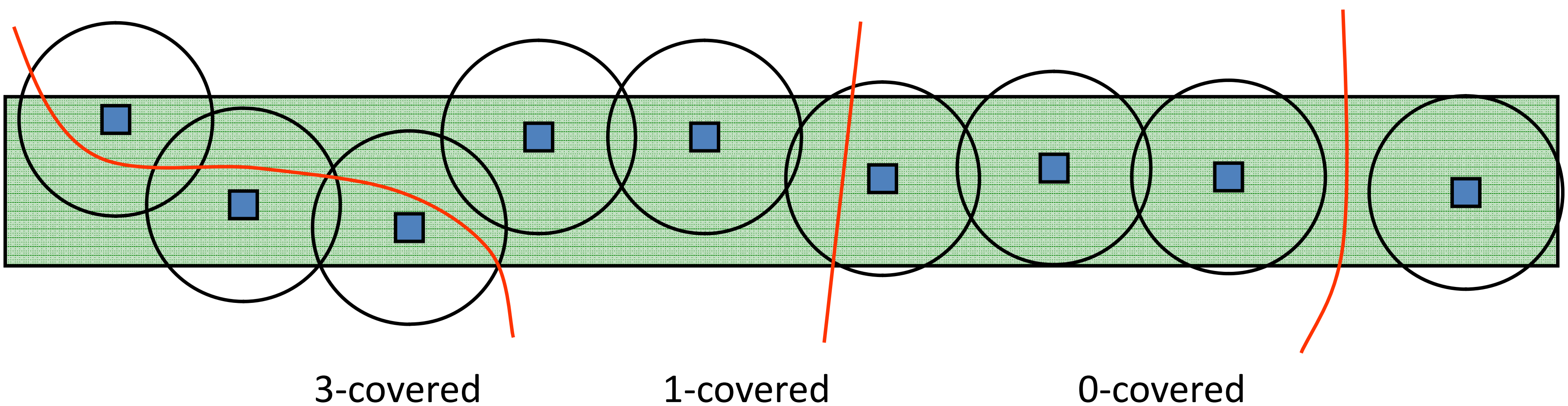
Crossing paths

Not crossing paths



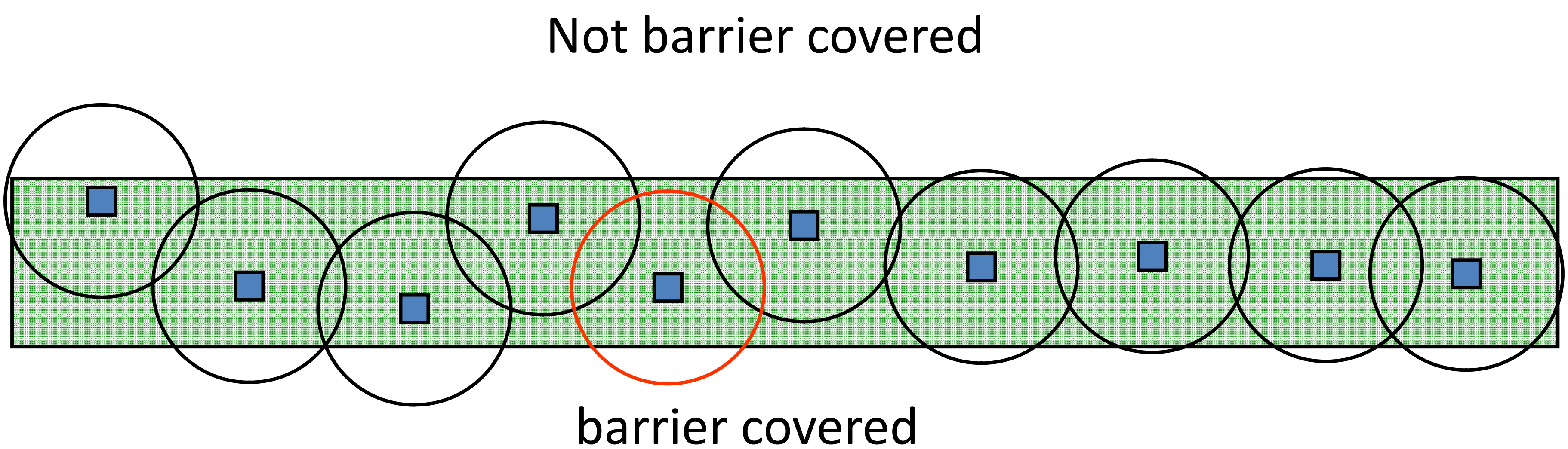
k-Coverage of a Path

- A crossing path is said to be *k*-covered if it intersects the sensing disks of at least *k* distinct sensors.



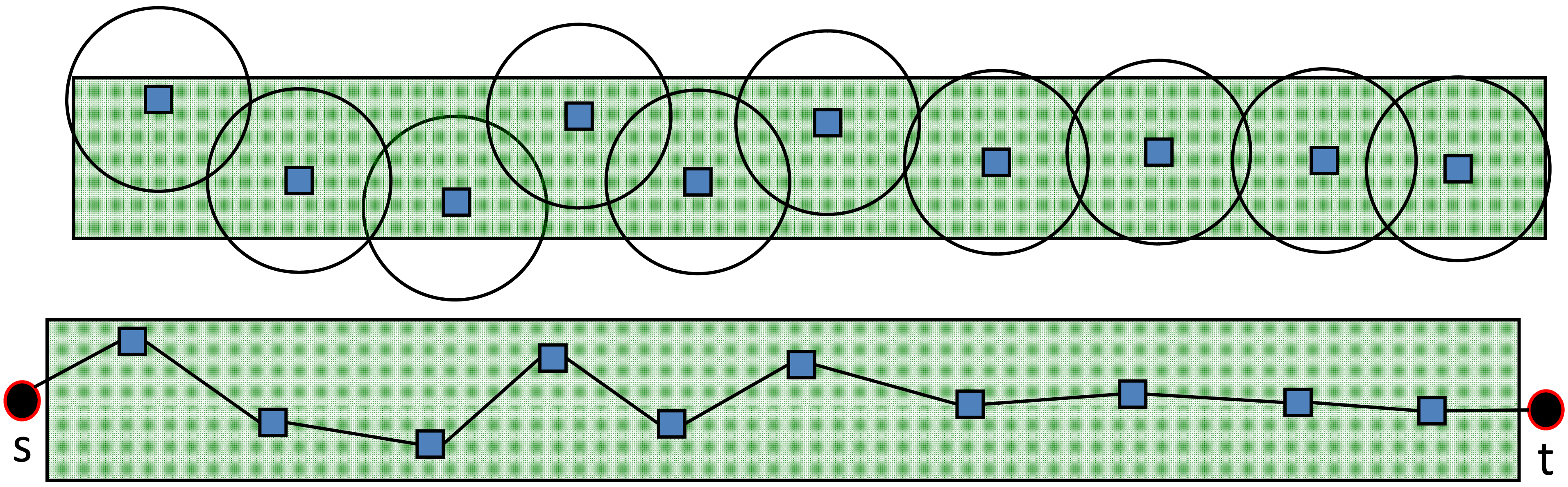
Barrier Coverage

- A belt region is barrier covered if all crossing paths are covered.



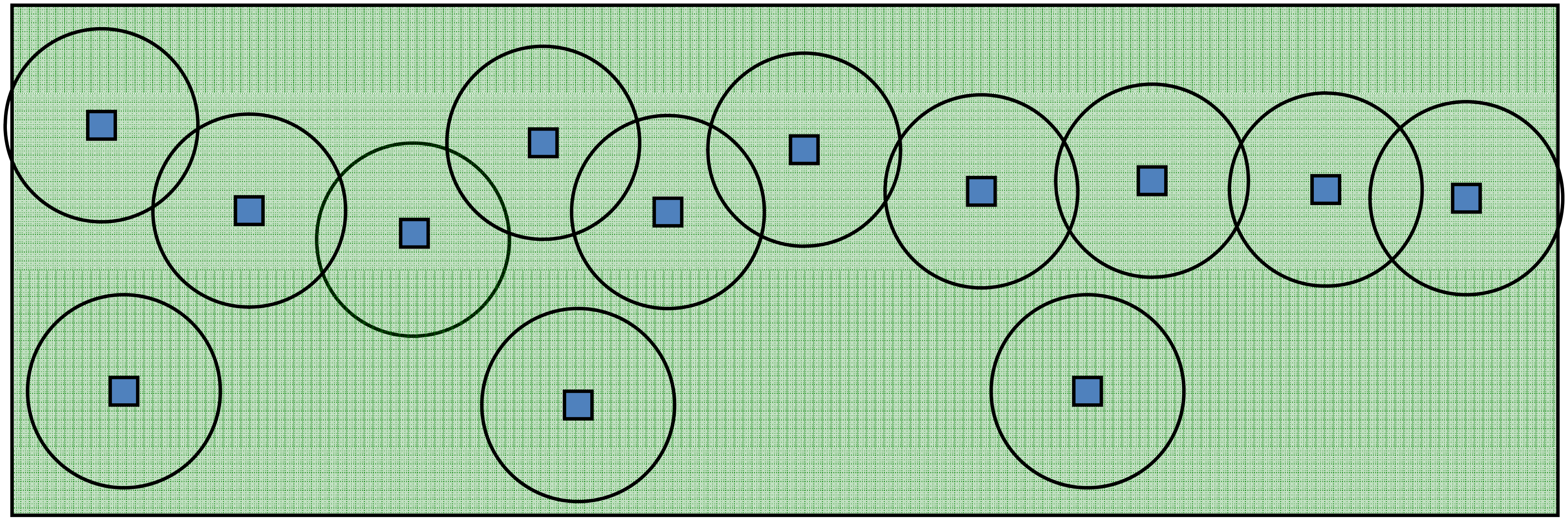
s-t Connectivity

- Same as barrier coverage
  - Use communication range in place of 2*sensing range



Full Connectivity

- All nodes need to be connected to each other



Coverage and Connectivity

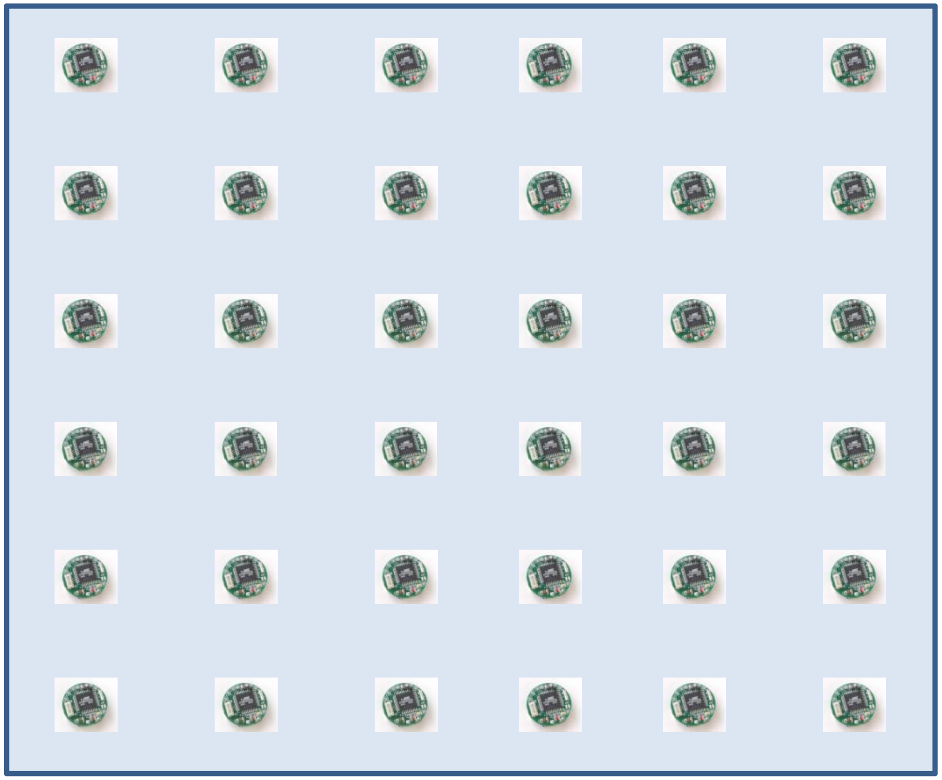
- Full Coverage
  - Every point in the region is covered
- Barrier Coverage
  - No uncovered crossing paths exist
- *s*-*t* connectivity (connectivity along the strip)
- Full Connectivity
  - All nodes in the region are connected to each other (possibly via multiple hops)

Outline

- Defining coverage and connectivity
- Model
- Asymptotic probabilistic condition and its limitations
- Reliable density estimates – introduction and its application to full connectivity
- Deriving reliable density estimate for barrier coverage in thin strips

Why consider random deployment

- Deployment errors
- Nature induced movements
- Unanticipated failures
- Poisson usually models worst case



The Parameters

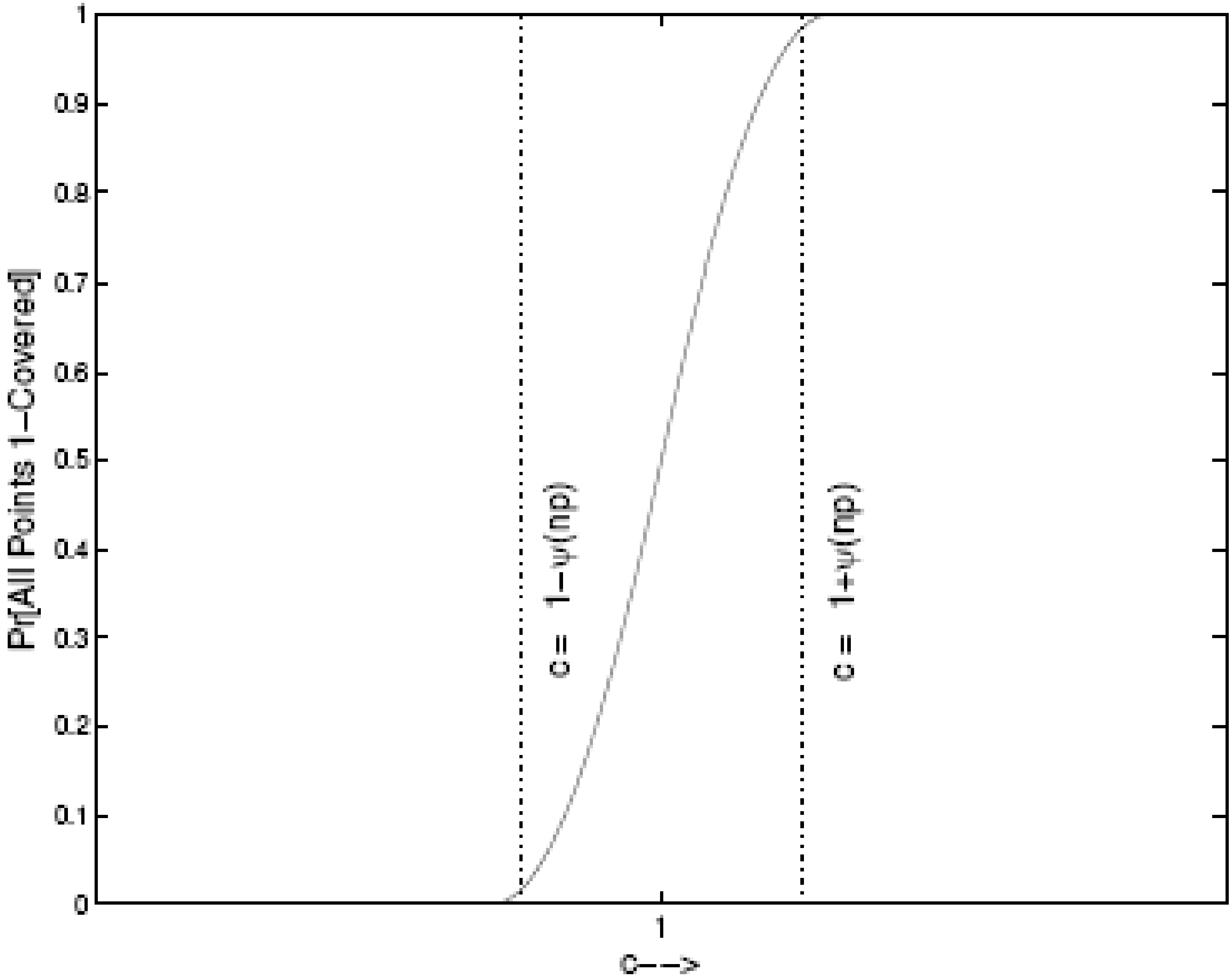
- For a given network,
- S – deployment region (rectangular)
  - ℓ - length
  - h - height
- λ – density of nodes
- r – *2**sensing radius or communication radius
- If the region size is scaled appropriately so that $\lambda = 1$, then r captures the variation in density

Outline

- Defining coverage and connectivity
- Model
- Asymptotic probabilistic condition and its limitations
- Reliable density estimates – introduction and its application to full connectivity
- Deriving reliable density estimate for barrier coverage in thin strips

Asymptotic Probabilistic Conditions

- Critical conditions (Phase Transition)
  - A sufficient condition that guarantees coverage with high probability
  - And, a sufficient condition that guarantees non-coverage with high probability
  - Both conditions converge to the same value at infinity



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Pr[All Points 1−Covered]
c = 1−ψ(np)
c = 1+ψ(np)
1
c−−>

Full Connectivity Example

- Full connectivity is asymptotically achieved
  - In a square (or disk) of area *n* (keeping *r* variable)
  - When $\pi r^2 = log\ n + c(n)$, iff $c(n) \rightarrow \infty$
- Observations
  - For a given finite region, what is an appropriate value of $c(n)$?
  - For a finite region and a given density (or radius), what is the probability of connectivity?
    - It is close to 1, but how close?

Outline

- Defining coverage and connectivity
- Model
- Asymptotic probabilistic condition and its limitations
- Reliable density estimates – introduction and its application to full connectivity
- Deriving reliable density estimate for barrier coverage in thin strips

The new paradigm

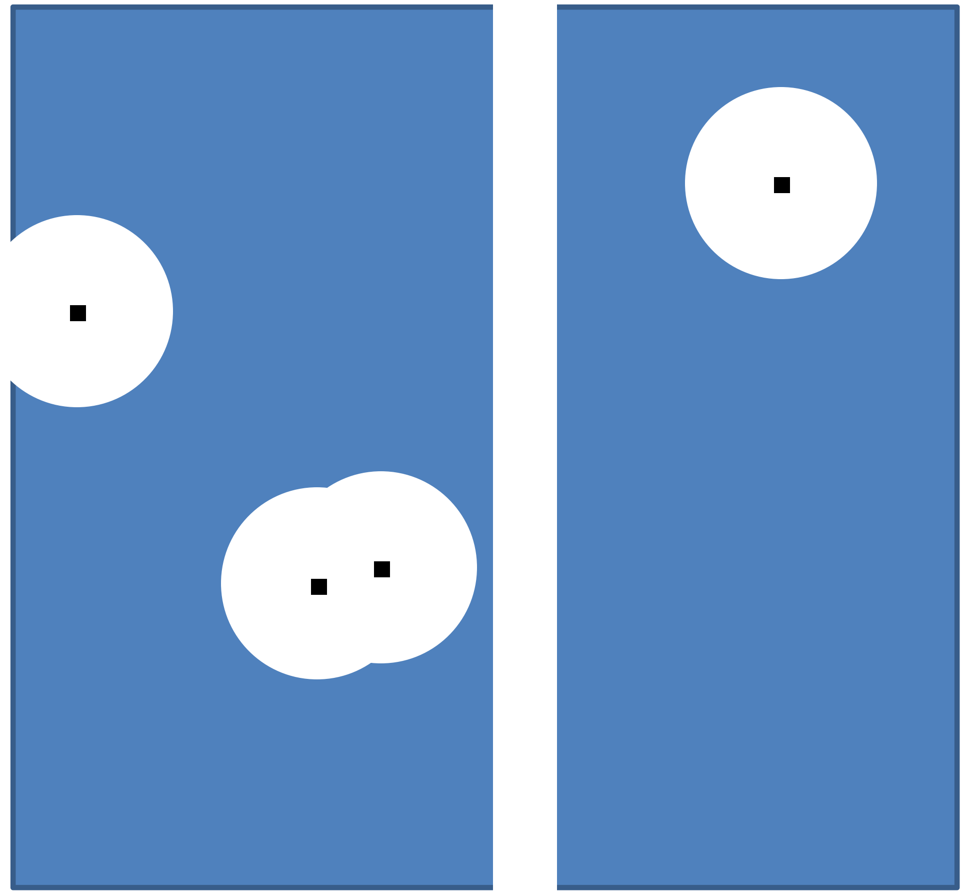
- Derive reliable probabilistic estimates
- Key steps/challenges
  - What is the key obstructing event?
  - Carefully define the event so its probability distribution can be derived
    - Ideal if the distribution is Poisson
  - Finally, estimate its parameters
    - Only intensity if the distribution is Poisson

How to find/guess the obstruction

- Characterizing the main excluded area
  - The smaller the region, higher the probability
  - More ways that this particular excluded area can occur, higher its probability
  - Can then observe using simulation the quality of approximation

Obstruction to Connectivity

- Several possible obstructions
- Which one dominates?



Deriving probability distribution

- Ideal if can prove Poisson distribution
- Need to show
  - The events (spatially) further apart are independent
  - The dependent event sets are negligible
    - Such dependent sets would occur even if the event was distributed according to a perfect Poisson distribution
- Formalize it by using the Stein –Chen Method for Poisson approximation

The Case of Connectivity Again

- Isolated nodes in spatially disjoint regions are independent (Poisson distributed nodes)
- The only dependent ones are those at most *2r* apart
  - whose probability of occurrence is much smaller as compared with that of isolated nodes

Estimating the parameter(s)

- If Poisson, then only the intensity (or expected number of excluded events) need to be derived
  - It is about the same as the probability of one such event occurring

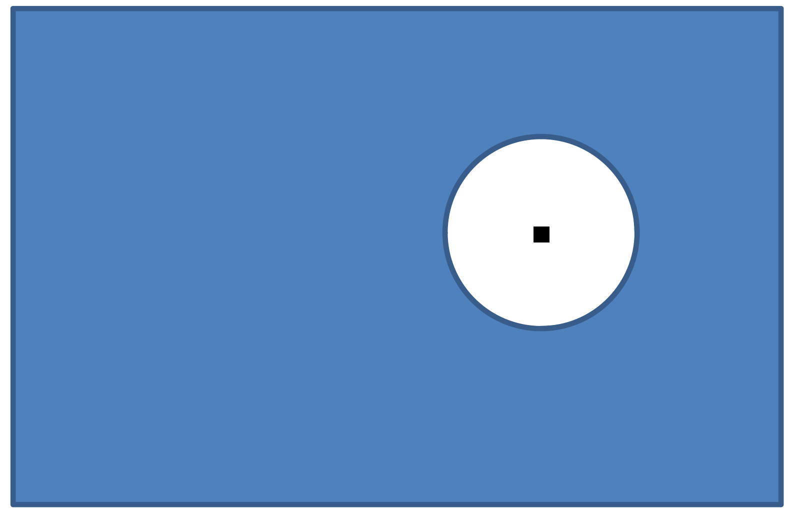
Parameter Estimation for Connectivity

- Probability of an arbitrary node being isolated
  - Or, a disk of radius *r* around this node being empty

$$e^{-\pi r^2}$$

- The probability of no isolated nodes occurring in area *n*

$$\approx e^{-ne^{-\pi r^2}}$$

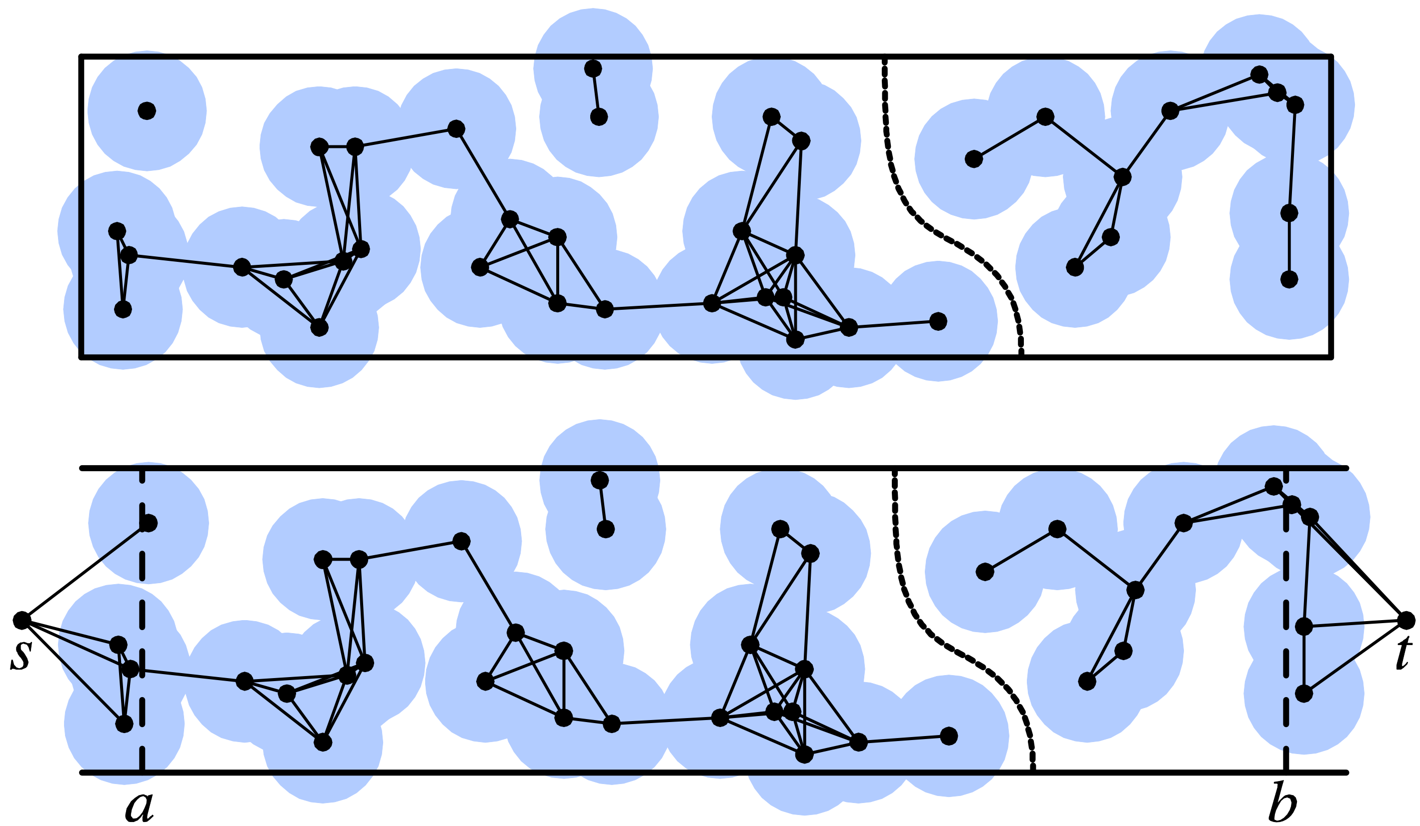


Outline

- Defining coverage and connectivity
- Model
- Percolation, critical density and their limitations
- Reliable density estimates – introduction and its application to full connectivity
- Deriving reliable density estimate for barrier coverage in thin strips

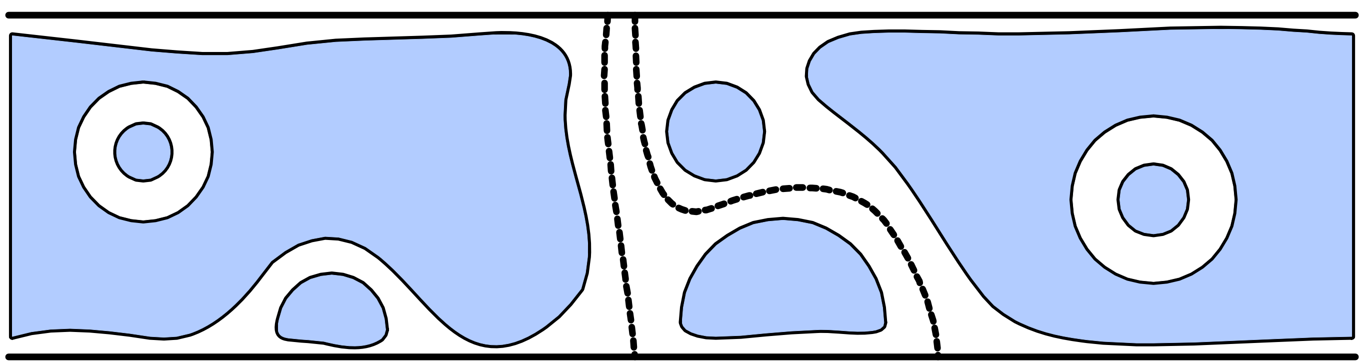
A More Challenging Case

- Barrier coverage (or *s*-*t* connectivity) in thin strips

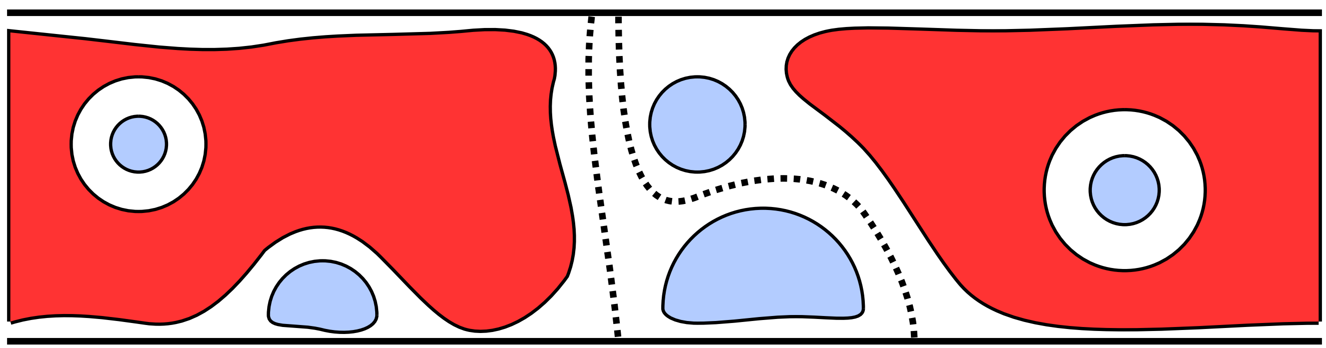


Key Challenges

- What is the main obstructing event?
  - Breaks
- How to characterize breaks so that we can
  - Prove their near independence
  - And, derive the probability of their occurrence



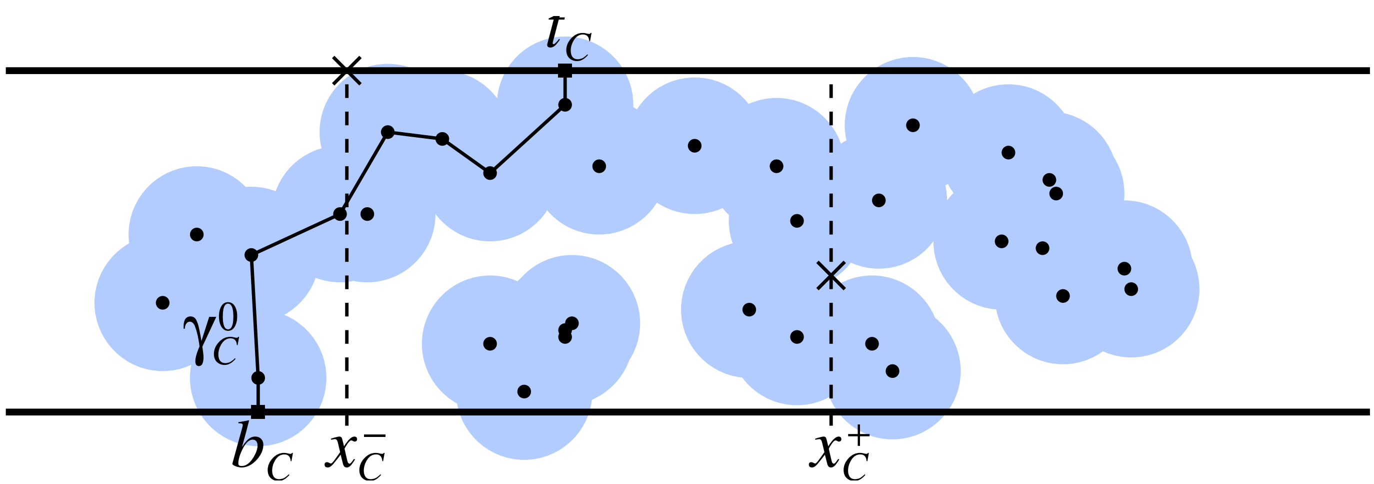
A Novel Definition



- Define a Good Component to be a graph component that has
  - Sensors within $\frac{r\sqrt{3}}{2}$ of both top and bottom
- Break is now the gap between consecutive good components

Implications

- No good component can sneak over or under another one
  - Good components are always separated by one and only one break
- Break ⇔ Barrier Coverage



Why are Breaks Poisson Distributed

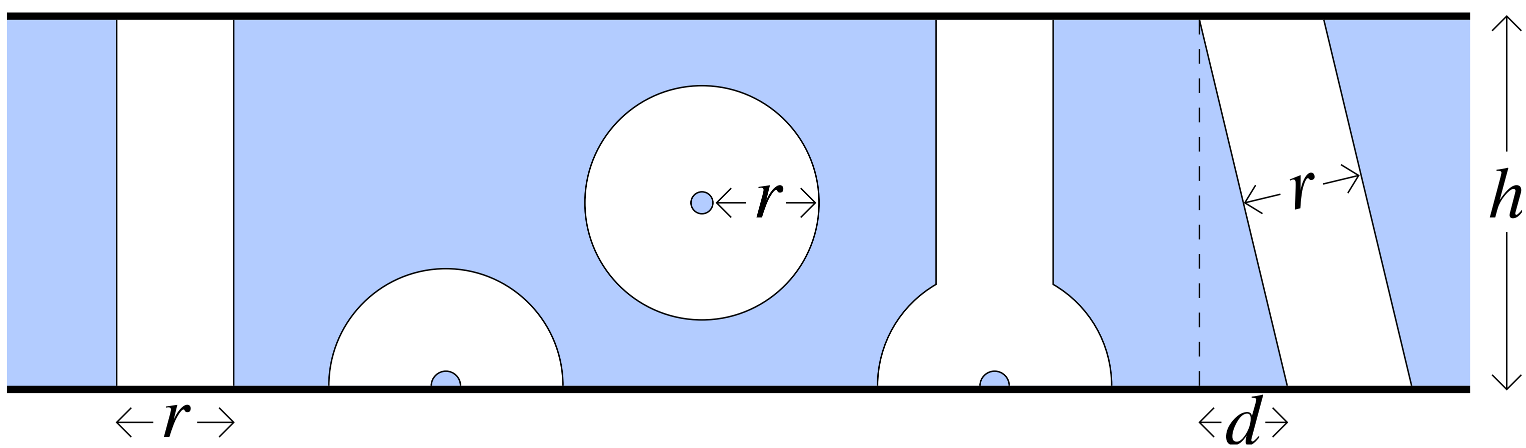
- Breaks are usually thin
  - If $r > 6$, then the average width of a break is at most $max\{5h, 1/h+2h\}$
- Good components are usually wide
  - If $r > 6$, then the proportion of good components with width less than $w > 0$ is at most $(w+7)e^{-h}$
- Hence, breaks are few and far between
- Can now apply Stein-Chen method to prove Poisson distribution of breaks

Estimating Break Intensity

- How do we identify a break so we can derive the probability of it occurring?
  - One-to-one mapping between breaks and good components
  - A good component can be identified by its right most node

Characterizing Break Shapes

- How does a break look like so we can estimate the vacant region following a good component?



The Final Estimate

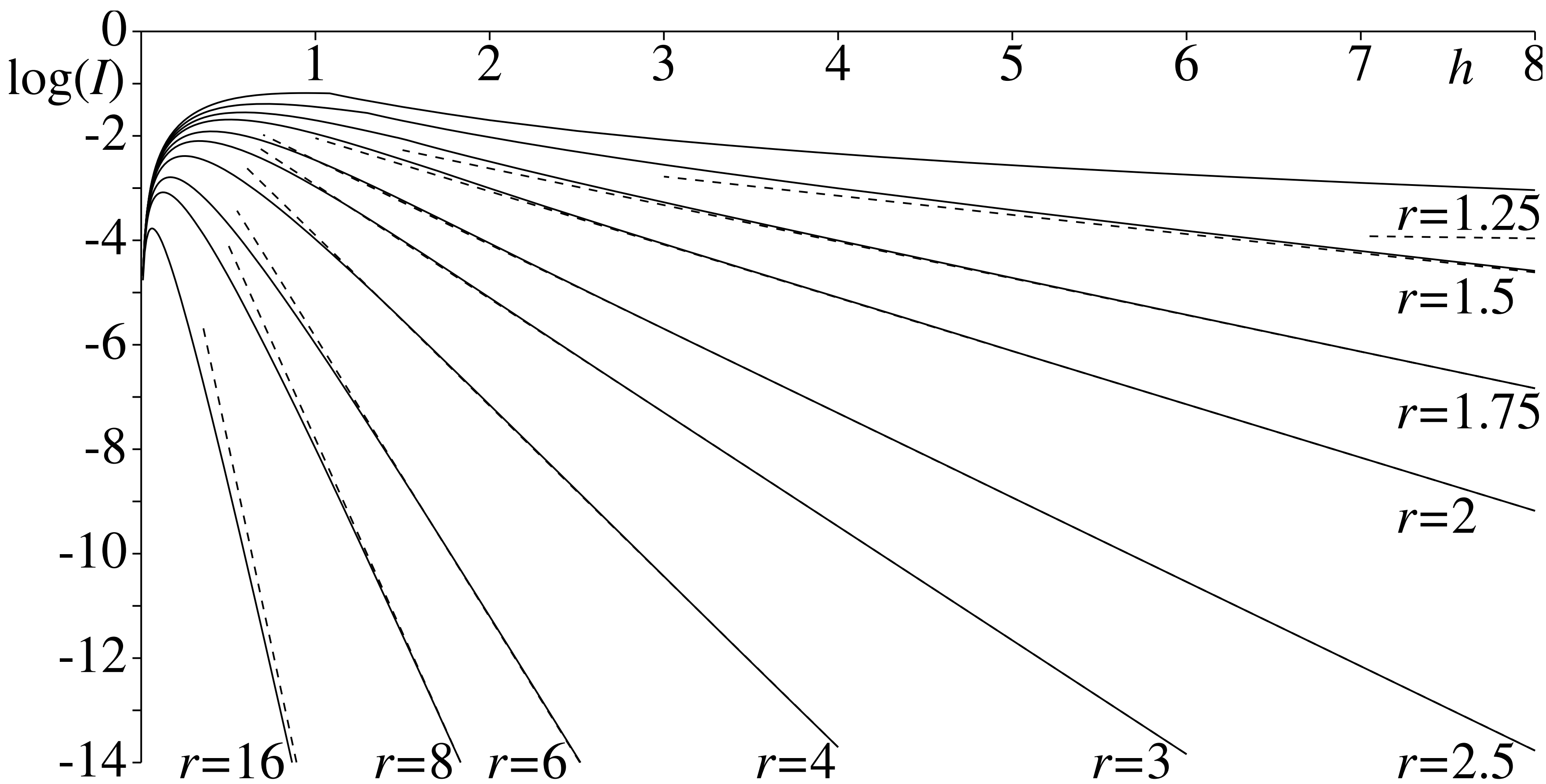
$$I_{h,r,\lambda} = \sqrt{\lambda}\,\exp\left(-\alpha_{r\sqrt{\lambda}} h\sqrt{\lambda} - \beta_{r\sqrt{\lambda}} + o(1)\right)$$

where

$$\alpha_x = x - 1.12794 x^{-1/3} - 0.2 x^{-5/3}$$

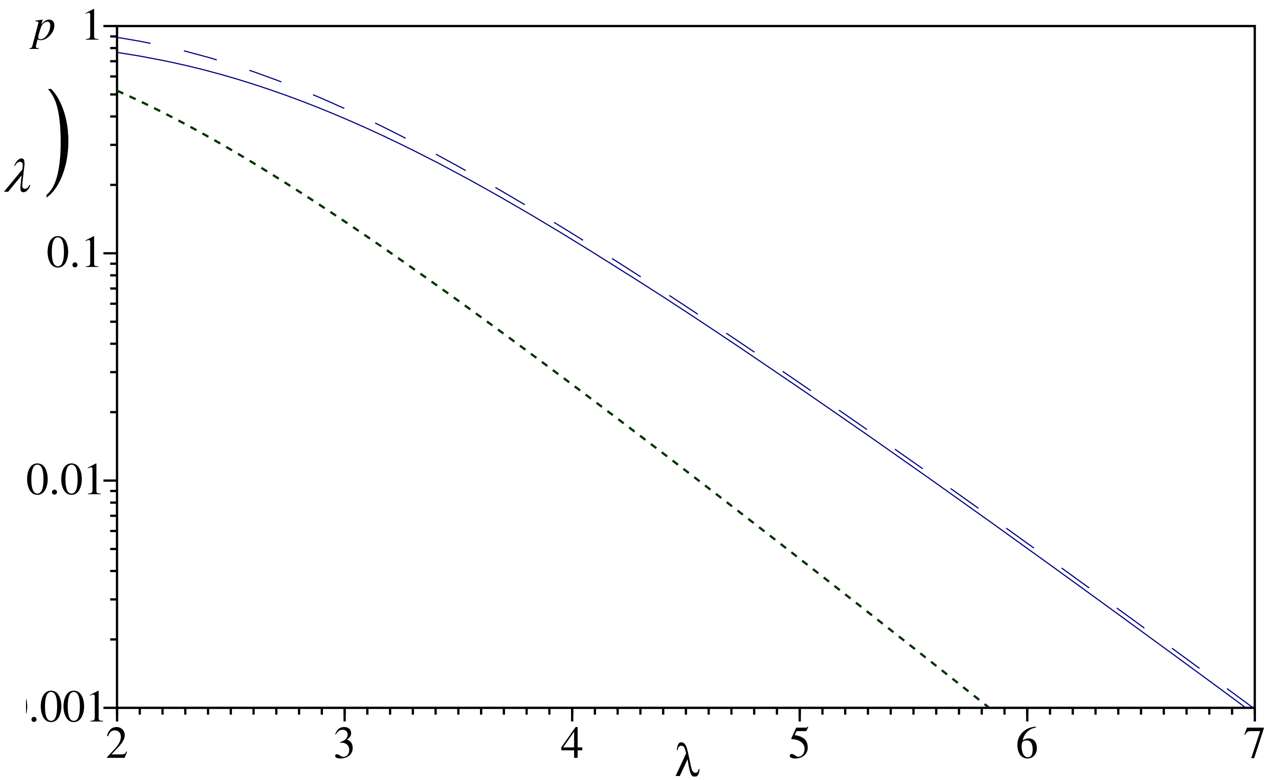
$$\beta_x = -\frac{1}{3}\log x + 1.05116 + 0.27 x^{-4/3}$$

How Good is Our Estimate



Probability of Break in a Finite Region

- Estimate
$$1-\exp\left(-\ell I_{h,r,\lambda}\right)$$
- $\ell = 10$
- $h = 2$
- $r = 1$



Conclusion

- Critical density (and percolation) has limited use in practice due to asymptotic nature
- Reliable probabilistic estimate can be derived in a systematic fashion
  - Need to carefully define the main obstruction
  - Derive its probability distribution
  - Estimate the intensity
- Showed that this approach is feasible even for a complex event, i.e., barrier coverage in thin strips

Looking Forward

- Can now envision theoretical results (in this area) being used in real deployments

Want to dig deeper

- Please refer to our paper
  - Paul Balister, Béla Bollobás, Amites Sarkar, and Santosh Kumar, "**Reliable Density Estimates for Coverage and Connectivity in Thin Strips of Finite Length**" in *ACM MobiCom 2007*
  - There is a draft of the journal version also available online that has detailed proofs