A New Metric for Space-Time Block Codes with Imperfect Channel Estimates

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***Abstract*— In this paper, a new metric for the decision rule of the space-time block (STB) codes having imperfect estimate of the channel fading parameters is derived. Estimation errors of these fading parameters have high impact on the performance of the STB code as the decision rule is dependent on these fading parameters. By including the variance of the channel estimation error in the decoder metric derivation of the STB codes, we derive an exact probability distribution function (pdf) of the received signals conditioned on the estimated channel parameters and transmitted symbol sequences. A modified decision rule is easily found from this and compared with the state-of-the-art scheme proposed by Tarokh. Performance comparison shows that significant improvement is achieved in terms of error rate and system complexity, especially for quadrature amplitude modulation (QAM). The derived decision rule converges to the ideal case for no estimation error and to Tarokh's decision rule for high signal-to-noise ratio (SNR).**

***Index Terms*— Space-time Block codes, imperfect channel estimates, wireless communications.**

I. INTRODUCTION

Simplicity of practical implementation and feasibility of having multiple antennas at the base station is the main reason to choose transmit diversity as a method of combating detrimental effects in the wireless communication system. While receive diversity like the maximal-ratio receiver combining (MRRC) scheme uses multiple receive antennas, the space-time block (STB) code uses multiple transit antennas to achieve performance gain. The advantage of STB code is that it allows us to achieve diversity gain while maintaining small physical size of the receiver. Historically, this scheme was first proposed by Alamouti [1] for two transmitter antennas and multiple receiver antennas. Later, the scheme was generalized for any number of transmitter and receiver antennas [12]–[14], [16], [19].

The derivation of the decision rule for decoding of the STB code, as given in [1], [14], assumes that the receiver has perfect knowledge of the channel state information (CSI). In practice, however, the receiver never has the perfect knowledge of the CSI, as the channel parameters are random variables. The parameters are estimated using a channel estimation technique as the decision rule requires the knowledge of these parameters. Thus all practical communication systems can be considered as systems with imperfect channel knowledge. For such STB coded systems with imperfect channel knowledge, if we employ the decision rule for perfect knowledge of CSI using imperfect channel parameter estimates in place of actual channel parameters, performance degradation of the whole system is observed due to mismatch. Performance degradation due to this type of mismatch in the channel parameters in the decision rule has been addressed in the standard literature [18]. It is shown in [2] that STB code is more sensitive to the channel estimation error than straightforward two branch diversity schemes, because of their dependency on the removal of the cross-terms in the decision rule. This dependency on the channel estimation error increases as the number of transmitter and receiver antenna increases to achieve high error performance [6].

To resolve this issue of mismatch due to the imperfect channel estimate, the case of partial knowledge of CSI was discussed and a modified decision rule was proposed by Tarokh [11], [13]. The partial knowledge of CSI utilized was the variance of the estimation error of the channel parameters, which can be easily and reliably obtained. Recently, however, it has been found that the scheme is only applicable when the SNR approaches infinity [15] and approximate for the practical range of SNR.

A systematic approach to include variance of the channel estimation error has been done by Frenger in [5] for Turbo coded systems. A similar approach is taken in this paper to calculate a new metric for a STB coded system proposed by Alamouti, which can easily be generalized for similar type of systems. A new modified decision rule has been derived which performs equal or supersedes Tarokh's decision rule with much less complexity. Estimation error variance of the pilot symbol channel estimation technique is also investigated.

The organization of this paper is as follows. Section II of this paper presents the system model including a channel estimation technique. The new decision metric for the STB code is derived from the exact pdf of the received signal conditioned on the estimated channel parameters in Section III. In Section IV, the proposed scheme is compared with the state-of-the-art scheme proposed by Tarokh. Simulation results are given in Section V. The conclusion and final comments are given in Section VI. The derivation of the exact conditional pdf of the received signal is given in the Appendix.

II. SYSTEM MODEL

A wireless communication system is considered with n transmitter antennas at the base station and one receiver antenna at the remote station. A simplified block diagram is given in Fig. 1. Extension of formulations for m receiver antennas is straightforward. Data is modulated in the modulator before encoding it with the STB encoder. The encoder

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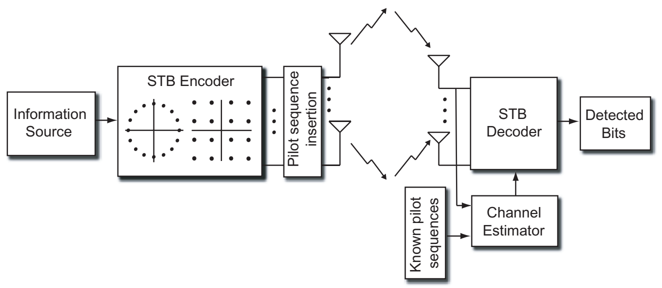

Fig. 1. Simplified block diagram of the STB coded communication system.

uses a generator matrix to encode the modulated data into different transmitter sequences, maintaining orthogonality of the sequences. Each of the n transmitter antennas simultaneously transmit one symbol $s_{i,t}, i = 0, 1, 2, \cdots, n-1$ at any time slot t. The received signals at the receiver are combined and then decoded using the maximum likelihood (ML) decoding algorithm. These signals are then demodulated to obtain the data bits.

We assume a flat fading wireless channel with the path gain defined to be h_i from the transmitter antenna i to the receiver antenna. The path gains are modelled as samples of zero mean, independent complex Gaussian random variables with the variance defined as $E[|h_i|^2] = 2\sigma_h^2$, where $E[x]$ denotes the expected value of x. Furthermore, the wireless channel is assumed to be quasi-static so that the path gains are constant over a frame of length L and vary from one frame to another. This incorporates the required assumption of the STB decoder to have constant fading for all the symbols of a block of transmitted symbol sequences having length of l. A frame consists of an integer number of blocks.

In this paper, we consider the STB coded system considered in [1] with 2 transmitter antennas and one receiver antenna. Data are encoded with STB encoder using the generator matrix

$$\mathbf{G} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (1)$$

where each column represents the signals transmitted from a particular antenna at different time slots and each row represents the signal vector transmitted from all transmitter antennas at a particular time slot. Here x^* denotes the complex conjugate of x.

After sampling of the received signal using the matched filter at the receiver, we have samples of the received signals [17]. Let the received signal at time slot 1 be r_1 and at time slot 2 be r_2, and the corresponding additive white Gaussian noise (AWGN) in the receiver be n_1 and n_2, respectively. So the received signal model can be written in vector form as

$$\mathbf{r} = \mathbf{G}\mathbf{h} + \mathbf{n} \quad (2)$$

where $\mathbf{r} = [r_1\ r_2]^T$, $\mathbf{h} = [h_1\ h_2]^T$ and $\mathbf{n} = [n_1\ n_2]^T$. Here $[\cdot]^T$ denotes transpose of the matrix or vector. Noise component $\mathbf{n}$ is a vector of complex valued Gaussian distributed elements with zero mean and the variance defined as $E[|n_1|^2] = E[|n_2|^2 = 2\sigma_n^2$. Actual fading parameter $\mathbf{h}$ is also a vector of complex valued Gaussian distributed elements with mean zero and the variance $E[|h_1|^2] = E[|h_2|^2 = 2\sigma_h^2$. All real and imaginary parts of $\mathbf{h}$ and $\mathbf{n}$ are assumed to be independent. It is straightforward to show that the variance of the sampled zero mean received signal becomes $E[|r_1|^2] = E[|r_2|^2] = 2\sigma_r^2 = 2\sigma_h^2(|s_1|^2 + |s_2|^2) + 2\sigma_n^2$.

Now, the estimated channel parameter model can be expressed as

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e} \quad (3)$$

where the estimated channel vector $\hat{\mathbf{h}} = [\hat{h}_1\ \hat{h}_2]^T$ and the error vector $\mathbf{e} = [e_1\ e_2]^T$. Here $\mathbf{e}$ is complex valued Gaussian distributed estimation error with $E[|e_1|^2] = E[|e_2|^2] = 2\sigma_e^2$ and $E[\mathbf{e}] = 0$. Assuming $\mathbf{h}$ and $\mathbf{e}$ to be independent, we find that $\hat{\mathbf{h}}$ is also a vector of complex valued Gaussian distributed random variables with zero mean and variance defined as $E[|\hat{h}_1|^2] = E[|\hat{h}_2|^2] = 2\sigma_{\hat{h}}^2 = 2\sigma_h^2 + 2\sigma_e^2$. It is shown in [3] that this channel estimate model is valid for pilot-based channel estimation schemes. Furthermore, in [4], this model is shown to hold for decision-directed channel estimation schemes, assuming that the previous data symbols used for channel estimation were correctly detected. This model can be used in other channel estimation models as well [8].

In this paper, we use a pilot based channel estimation technique, where the channel fading coefficients are estimated by inserting orthogonal pilot sequences in the transmitted signals. In this method, some pilot sequences are inserted at the beginning (or middle) of a data frame. The receiver has perfect knowledge of the positions and magnitudes of the pilot sequences. For multiple transmit antennas, the pilot sequence of any transmitter antenna must be orthogonal to other pilot sequences from other transmitter antennas to simplify channel estimator structure. For a system with n transmitter antennas, n different pilot sequences $P_1, P_2, ..., P_n$ with the same length are needed. Let k be the length of the pilot sequences, i.e., $P_i = [P_{i,1}\ P_{i,2}\ ...\ P_{i,k}]^T$ for the ith transmitter. To satisfy the orthogonality property, the pilot sequence of the ith transmitter has to satisfy the condition

$$P_i^T \cdot P_j^* = \begin{cases} 0 & \text{for } i \neq j \\ \|P_i\|^2 & \text{for } i = j \end{cases}$$

where j is any other transmitter antenna.

The receiver isolates the received signals due to the pilot symbols and sends those to the channel estimator for initial estimation of channel before decoding of the received signals due to data symbols. During the channel estimation, the received signal at the the receiver antenna at time t can be represented by

$$r_t = \sum_{i=1}^{n} h_i \cdot P_{i,t} + n_t. \quad (4)$$

The received signal and noise sequence at the antenna can be represented as $r_p = [r_1\ r_2\ ...\ r_k]^T$ and $n_p = [n_1\ n_2\ ...\ n_k]^T$, respectively. The receiver estimates the channel fading parameter h_i by using the observed sequences r_p. Since the pilot sequences $P_1, P_2, ..., P_n$ are orthogonal, the minimum mean square error (MMSE) estimate of h_i is given by [13]

$$\begin{aligned}\hat{h}_i &= r_p^T \cdot P_i^* / \|P_i\|^2 \\ &= h_i + (n_p^T \cdot P_i^*)/\|P_i\|^2 = h_i + e_i \end{aligned} \quad (5)$$

$$p_{r|\hat{h},s}(R|\hat{H},S) = \frac{1}{(2\pi)^2\sigma_r^4\left(1-|\mu|^2\right)^2}\exp\left[\frac{-1}{2\sigma_r^2\left(1-|\mu|^2\right)}\left(\left|r_1-\left(\mu_{11}\hat{h}_1+\mu_{12}\hat{h}_2\right)\frac{\sigma_r}{\sigma_h}\right|^2 + \left|r_2-\left(\mu_{11}^*\hat{h}_2-\mu_{12}^*\hat{h}_1\right)\frac{\sigma_r}{\sigma_h}\right|^2\right)\right] \tag{10}$$

$$\begin{aligned} p_{r|\hat{h},s}(R|\hat{H},S) = & \frac{1}{2\pi\sigma_r^2\left(1-|\mu|^2\right)}\exp\left\{-\frac{1}{2\sigma_r^2\left(1-|\mu|^2\right)}\left|r_1-\left(\mu_{11}\hat{h}_1+\mu_{12}\hat{h}_2\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2\right\} \\ & \times\frac{1}{2\pi\sigma_r^2\left(1-|\mu|^2\right)}\exp\left\{-\frac{1}{2\sigma_r^2\left(1-|\mu|^2\right)}\left|r_2-\left(\mu_{11}^*\hat{h}_2-\mu_{12}^*\hat{h}_1\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2\right\} \end{aligned} \tag{11}$$

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where e_i is the estimation error due to the noise, given by

$$e_i = (n_p^T \cdot P_i^*)/\|P_i\|^2. \tag{6}$$

Since n_p is a zero-mean complex Gaussian random variable with single-sided power spectral density N_0, the estimation error e_i has a zero mean and single-sided power spectral density N_0/k [13].

In order to derive the modified decision metric, we need to know the exact pdf of the received signal conditioned on the estimated channel parameters and transmitted symbol sequences. To simplify our calculations, the following cross correlation coefficients are defined:

$$\mu_{11} = \frac{E[r_1 h_1^*]}{\sqrt{\mathrm{var}(r_1)\mathrm{var}(h_1)}} \quad \mu_{12} = \frac{E[r_1 h_2^*]}{\sqrt{\mathrm{var}(r_1)\mathrm{var}(h_2)}}$$
$$\mu_{21} = \frac{E[r_2 h_1^*]}{\sqrt{\mathrm{var}(r_2)\mathrm{var}(h_1)}} \quad \mu_{22} = \frac{E[r_2 h_2^*]}{\sqrt{\mathrm{var}(r_2)\mathrm{var}(h_2)}}.$$

It can be easily shown that $\mu_{11} = \mu_{22}^* = s_1\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$ and $\mu_{12} = -\mu_{21}^* = s_2\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$. We further define

$$|\mu|^2 = |\mu_{ij}|^2 + |\mu_{mn}|^2 = (|s_1|^2+|s_2|^2)\frac{\sigma_h^4}{\sigma_r^2\sigma_{\hat{h}}^2} \tag{7}$$

where $i, j, m, n \in \{1, 2\}$ with the condition that if $i = m$, then $j \neq n$ or vice versa.

As shown in the Appendix, the required pdf can be expressed as given in (10), where s is the vector of signals transmitted at a particular time slot.

III. Derivation of The Decision Metric

We can easily find that the pdf described in (10) can be expressed as multiplication of two pdf's, as shown in (11), which is in the form of

$$p_{r|\hat{h},s}(R|\hat{H},S) = p_{r_1|\hat{h},s}(R_1|\hat{H},S)p_{r_2|\hat{h},s}(R_2|\hat{H},S). \tag{12}$$

It is obvious that the conditional distributions of r_1 and r_2 are independent, Gaussian distributed with conditional expected values of

$$E\left[\left|r_1|\hat{h},s\right|\right] = (\mu_{11}\hat{h}_1+\mu_{12}\hat{h}_2)\frac{\sigma_r}{\sigma_{\hat{h}}}, \tag{13}$$

$$E\left[\left|r_2|\hat{h},s\right|\right] = (\mu_{11}^*\hat{h}_2-\mu_{12}^*\hat{h}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}. \tag{14}$$

The conditional variances are equal and given as

$$E\left[\left|r_1|\hat{h},s\right|^2\right] = E\left[\left|r_2|\hat{h},s\right|^2\right] = 2\sigma_r^2(1-|\mu|^2). \tag{15}$$

Assuming that all the signals in the modulation constellation are equiprobable, a ML decoder chooses a pair of signals from the signal modulation constellation to minimize the distance metric

$$d^2(r_1, E[\left|r_1|\hat{h},\hat{s}\right|]) + d^2(r_2, E[\left|r_2|\hat{h},\hat{s}\right|]) \tag{16}$$

over all possible values of $\hat{s}$, the detected signal sequence vector. Here $d^2(x, y)$ is the squared Euclidean distance between signals x and y calculated using the expression $d^2(x, y) = (x - y)(x^* - y^*)$.

Putting values in (16) leads us to the minimization problem of the following distance metric

$$\begin{aligned} &|r_1-(\mu_{11}\hat{h}_1+\mu_{12}\hat{h}_2)\frac{\sigma_r}{\sigma_{\hat{h}}}|^2 \\ &+|r_2-(\mu_{11}^*\hat{h}_2-\mu_{12}^*\hat{h}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}|^2 \end{aligned} \tag{17}$$

for all transmitted symbol sequences.

After expanding the above metric and deleting the terms independent of the transmitted symbols, we reach the following equivalent metric to be minimized

$$\begin{aligned} &\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}(-r_1\hat{h}_1^*s_1^*-r_1^*\hat{h}_1s_1-r_1\hat{h}_2^*s_2^*-r_1^*\hat{h}_2s_2-r_2\hat{h}_2^*s_1 \\ &-r_2^*\hat{h}_2s_1^*+r_2\hat{h}_1^*s_2+r_2^*\hat{h}_1s_2^*)+\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|s_1|^2|\hat{h}_1|^2 \\ &+|s_2|^2|\hat{h}_2|^2+|s_1|^2|\hat{h}_2|^2+|s_2|^2|\hat{h}_1|^2). \end{aligned}$$

We can decompose this term into two parts for the sake of the simplicity of the detection process, one of which

$$\begin{aligned} &\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left(-(r_1\hat{h}_1^*+r_2^*\hat{h}_2)s_1^*-(r_1^*\hat{h}_1+r_2\hat{h}_2^*)s_1\right) \\ &+\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{h}_1|^2+|\hat{h}_2|^2\right)|s_1|^2 \end{aligned}$$

is a function of s_1 only, and the other one

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left(-(r_1\hat{h}_2^* - r_2^*\hat{h}_1)s_2^* - (r_1^*\hat{h}_2 - r_2\hat{h}_1^*)s_2\right) + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{h}_1|^2 + |\hat{h}_2|^2\right)|s_2|^2$$

is a function of s_2 only. Thus the minimization problem given in (17) is reduced to minimizing these two parts separately. This leads us to a faster decoding process with less complexity, especially with higher order modulation schemes. After some rearrangement and manipulation of the above two expressions, we reach the decision metric

$$|(r_1\hat{h}_1^* + r_2^*\hat{h}_2)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|\hat{h}_1|^2 + |\hat{h}_2|^2))|s_1|^2 \quad (18)$$

for detecting s_1 and the decision metric

$$|(r_1\hat{h}_2^* - r_2^*\hat{h}_1)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_2|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|\hat{h}_1|^2 + |\hat{h}_2|^2))|s_2|^2 \quad (19)$$

for detecting s_2. These are the desired modified decision rules and are used in our simulations for a STB coded system with the imperfect channel estimates.

For the ideal case in which we have perfect knowledge of CSI at the receiver, hence no estimation error, we have $\sigma_h^2 = \sigma_{\hat{h}}^2$ as $\sigma_e^2 = 0$ and $h_i = \hat{h}_i$, where $i \in \{1, 2\}$. Consequently (18) and (19) becomes the same as the decision rules for the perfect channel knowledge, which are

$$|(r_1h_1^* + r_2^*h_2) - s_1|^2 + (-1 + (|h_1|^2 + |h_2|^2))|s_1|^2$$

for detecting s_1 and

$$|(r_1h_2^* - r_2^*h_1) - s_1|^2 + (-1 + (|h_1|^2 + |h_2|^2))|s_2|^2$$

for detection s_2 as given in [14].

IV. Comparison with Tarokh's Decision Metric

We now compare the proposed metric with the state-of-the-art metric given by Tarokh [11], [13] in the presence of the channel estimation error. Originally derived for the space-time trellis codes, the metric is however generally accepted for the STB codes. The mean and variance of the distribution function of the random variable r_t conditioned on $\hat{h}_i$ as given by Tarokh are $\mu_{r'} = \mu'/(\sqrt{2}\sigma_{\hat{h}})\sqrt{E_s}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i$ and $\sigma_{r'}^2 = N_0 + (1 - |\mu'|^2)E_s\sum_{i=1}^{n}\left|\hat{s}_{i,t}^2\right|$, respectively. Here $\mu' = 1/\sqrt{1 + 2\sigma_e^2}$ is the correlation coefficient, $N_0 = 2\sigma_n^2$ is the noise variance and E_s is the energy per symbol, which is the factor by which the elements of the signal constellation are contracted to make the average energy of the constellation as 1. The decision metric proposed by Tarokh for n transmitter antennas and one receiver antenna can be written as

$$\sum_{t=1}^{l}\left(\frac{\left|r_t - \frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2}{N_0 + (1 - |\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2} + \ln(N_0 + (1 - |\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2)\right). \quad (20)$$

For the case of PSK (phase shift keying) constellation, the metric given in (20) is reduced to the following:

$$\sum_{t=1}^{l}\left|r_t - \frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2. \quad (21)$$

According to a recent publication of Tarokh, these expressions are only valid for very high SNR [15]. For SNR of infinite, $N_0/E_s \to 0$, i.e. for a certain E_s, $N_0 \to 0$. Again, for the case of normalized Rayleigh fading channel, which is assumed by Tarokh, we have $\sigma_h^2 = 0.5$ and $\sqrt{E_s} = 1$. Considering the case of 2 transmitter and 1 receiver antenna, the mean of the pdf of the received data conditioned on the estimated channel parameters becomes

$$\mu_{r'} = \frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i = \frac{1}{\sqrt{1 + 2\sigma_e^2}\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i = \frac{1}{2\sigma_{\hat{h}}^2}(\hat{s}_1\hat{h}_1 + \hat{s}_2\hat{h}_2) \quad (22)$$

Assuming correct detection, for r_1, the mean becomes

$$\mu_{r_1'} = \frac{1}{2\sigma_{\hat{h}}^2}(s_1\hat{h}_1 + s_2\hat{h}_2) = \frac{\sigma_r}{\sigma_{\hat{h}}}(\frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_1\hat{h}_1 + \frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_2\hat{h}_2) = \frac{\sigma_r}{\sigma_{\hat{h}}}(\mu_{00}\hat{h}_1 + \mu_{01}\hat{h}_2) \quad (23)$$

which is equal to the mean given in (13). Similarly, for r_2, the mean becomes

$$\mu_{r_2'} = \frac{\sigma_r}{\sigma_{\hat{h}}}(\mu_{00}^*\hat{h}_2 - \mu_{01}^*\hat{h}_1) \quad (24)$$

which is equal to the mean given in (14). For the same conditions, the variance of the pdf of the received data conditioned on the estimated channel parameters becomes

$$\sigma_{r'}^2 = (1 - \frac{1}{1 + 2\sigma_e^2})\sum_{i=1}^{2}|\hat{s}_i|^2 = \frac{2\sigma_e^2}{2\sigma_{\hat{h}}^2}\sum_{i=1}^{2}|\hat{s}_i|^2 = \frac{\sigma_e^2}{\sigma_{\hat{h}}^2}(|\hat{s}_1|^2 + |\hat{s}_2|^2) \quad (25)$$

for both r_1 and r_2. The variance of the received signal conditioned on the estimated channel parameter derived in this paper is given in (15). Expanding this variance with the conditions that $\sigma_n^2 \to 0$ as SNR goes to infinity i.e.

$\sigma_r^2 = \sigma_h^2(|s_1|^2+|s_2|^2)+\sigma_n^2 \rightarrow \sigma_h^2(|s_1|^2+|s_2|^2)$ and $\sigma_h^2 = 0.5$, we have

$$2\sigma_r^2(1-|\mu|^2) = (|s_1|^2+|s_2|^2)(1-\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}) = (|s_1|^2+|s_2|^2)\frac{\sigma_e^2}{\sigma_{\hat{h}}^2} \quad (26)$$

which is exactly equal to (25). Hence it is found that the proposed scheme becomes the same as the scheme proposed by Tarokh for high SNR.

To address the complexity issues and memory requirements of the proposed scheme compared with that of Tarokh's scheme, some assumptions are made in order to simplify our comparative study. In this simplified approach, we are interested only in the complexity of the decision rule as the difference of the two schemes lies in the decision rules. Since multiplications and divisions are the most complex basic operations in designing the signal processors, we base our calculations on the requirements of these operations in the decision rules. We assume the complexity of a multiplier, a divisor, a squaring or a logarithm operation be the same. The complexity index is calculated by simply adding the required number of multiplications, divisions, squaring and logarithm operations. Table I gives the comparison of the proposed scheme with Tarokh's scheme for p-QAM and p-PSK modulation, where p is the number of states of the modulation scheme. The metrics are to be computed for each possible combination of the signals transmitted for each state of the modulation scheme. Assume that factors are to be calculated once and then stored for reuse in the later computations, rather than calculating each time. This results in minimized processing time by reducing the number of multiplications with increasing storage size. The metrics proposed by Tarokh have to be computed p^2 times for different combinations of $\hat{s}_1$ and $\hat{s}_2$ and compared with each other. However, in the proposed scheme, each metric has to be computed only p times because of the variable separation operation during the derivation of the metrics. This is why the complexity of the decision rule of the proposed scheme is much lower than that of Tarokh's scheme.

V. Simulation Results

The simulations have been performed using Alamouti's scheme of the STB codes with two transmitter and one receiver antenna. The channel is assumed to be quasi-static and flat faded. Normalized Rayleigh fading is assumed with the variance per complex dimension as $\sigma_h^2 = 0.5$. For pilot symbol insertion, the total transmit energy per frame is kept constant for a fair comparison.

The channel estimation error variance σ_e^2 depends on the actual channel estimation scheme and can be computed as a function of bit energy-to-noise ratio and the number of pilot symbols [13]. To find these variances of estimation error for different lengths of pilot symbols, we have transmitted only pilot symbols from the transmitters and estimated the channel using (5) and plotted it against a range of E_b/N_0 as shown in Fig. 2, where E_b is the bit energy and N_0 is the noise variance. The estimation error is found to decrease with increasing signal power and increasing length of pilot symbols. Theoretically the estimation error variances can be found from the expression $N_0/2kE_s$ per dimension, where k is the length of the pilot sequence. The theoretical results are found to be virtually identical to that of the experimental values. However, the experimental values of the estimation error variances are used in the later simulations which is obtained from a look-up table. This is done for better resemblance to the practical implementation issues.

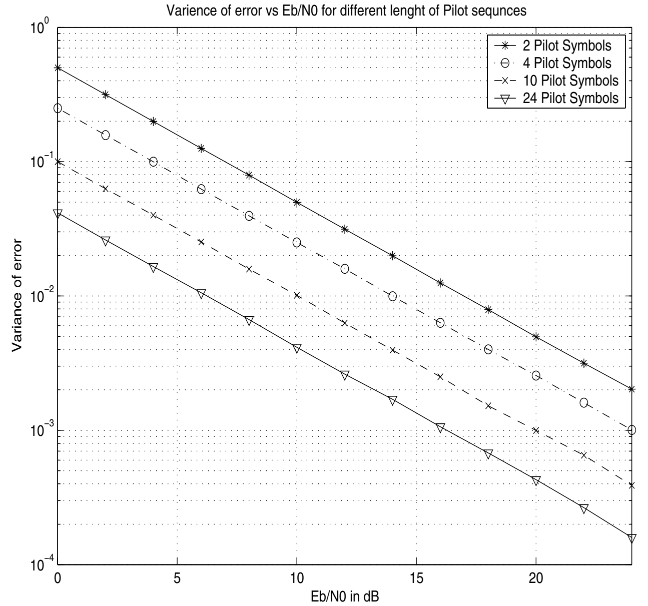

Fig. 2. Variance of estimation error vs E_b/N_0 for different lengths of pilot sequences in slow Rayleigh fading channel.

The transmitted signal sequences are modulated using a 16-PSK and a gray coded 16-QAM modulation scheme [10]. Experimental results show that the gain in error rate with respect to Tarokh's model is insignificant for PSK modulation. However, for QAM modulation, significant gain is observed. Here the results for the frame length of 128 bits/frame and 512 bits/frame with 16-QAM gray coded modulation are presented in Fig. 3 and 4 respectively. Two pilot symbols are added to each frame for the channel estimation. As observed, the gain using the proposed method is higher with higher number of bits per frame.

In Fig. 5, the gain in dB is plotted against the BER. It shows that the gain remains high for low BER with larger frame length. So it can be predicted that the gain will be substantial for practical range of frame size and error rate.

VI. Conclusion

In this paper, an exact pdf of the received signal conditioned on the estimated channel parameters and the transmitted symbol sequences is derived. Using this pdf, we find a new modified decision rule for the decoding of the STB coded system with partial knowledge of CSI. Simulation results show that there is significant performance improvement over the state-of-the-art method in terms of error rate and system complexity. The proposed scheme performs especially well for

Modulation Scheme	Decision Metric	Complexity Index	Memory Requirements	Number of Comparisons
p-QAM	Proposed (18) (19)	$17\times p$	10	$p(p-1)$
p-QAM	Tarokh (20)	$20\times p^2$	13	$p^2(p^2-1)/2$
p-PSK	Proposed (18) (19)	$17\times p$	10	$p(p-1)$
p-PSK	Tarokh (21)	$11\times p^2$	11	$p^2(p^2-1)/2$

TABLE I
COMPARISON OF THE COMPLEXITY ISSUES OF THE PROPOSED SCHEME WITH TAROKH'S SCHEME.

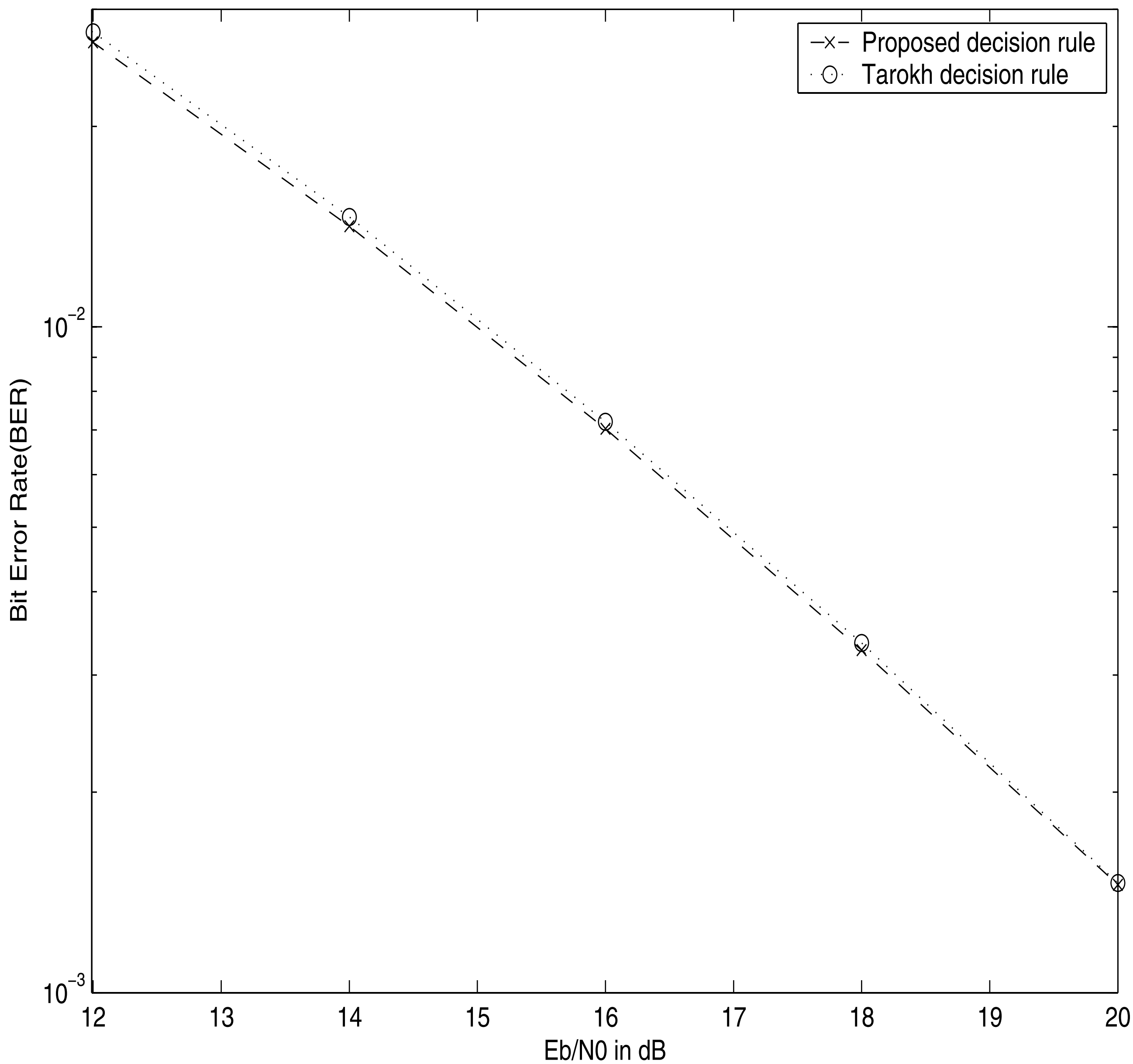

Fig. 3. BER performance for 16-QAM gray coded modulation (128 bits/frame) with the STB coded system .

QAM modulation, which is commonly found in the practical systems. Simulation results are obtained using parameters estimated from the channel, not from the theoretical values of the channel estimation error. Hence the performance observed incorporates the degradation due to the of the variance of the estimation error of the pilot symbol sequences.

We derive the modified decision rule for the special case of 2 transmitter and 1 receiver antenna with STB coded communication channel proposed by Alamouti. It is straightforward to extend it for a higher number of transmitter and receiver antennas using the same approach. Hence a generalized modified decision rule can be obtained easily to be used for any STB coded system. The extremely low complexity of the proposed scheme makes it an attractive solution for practical

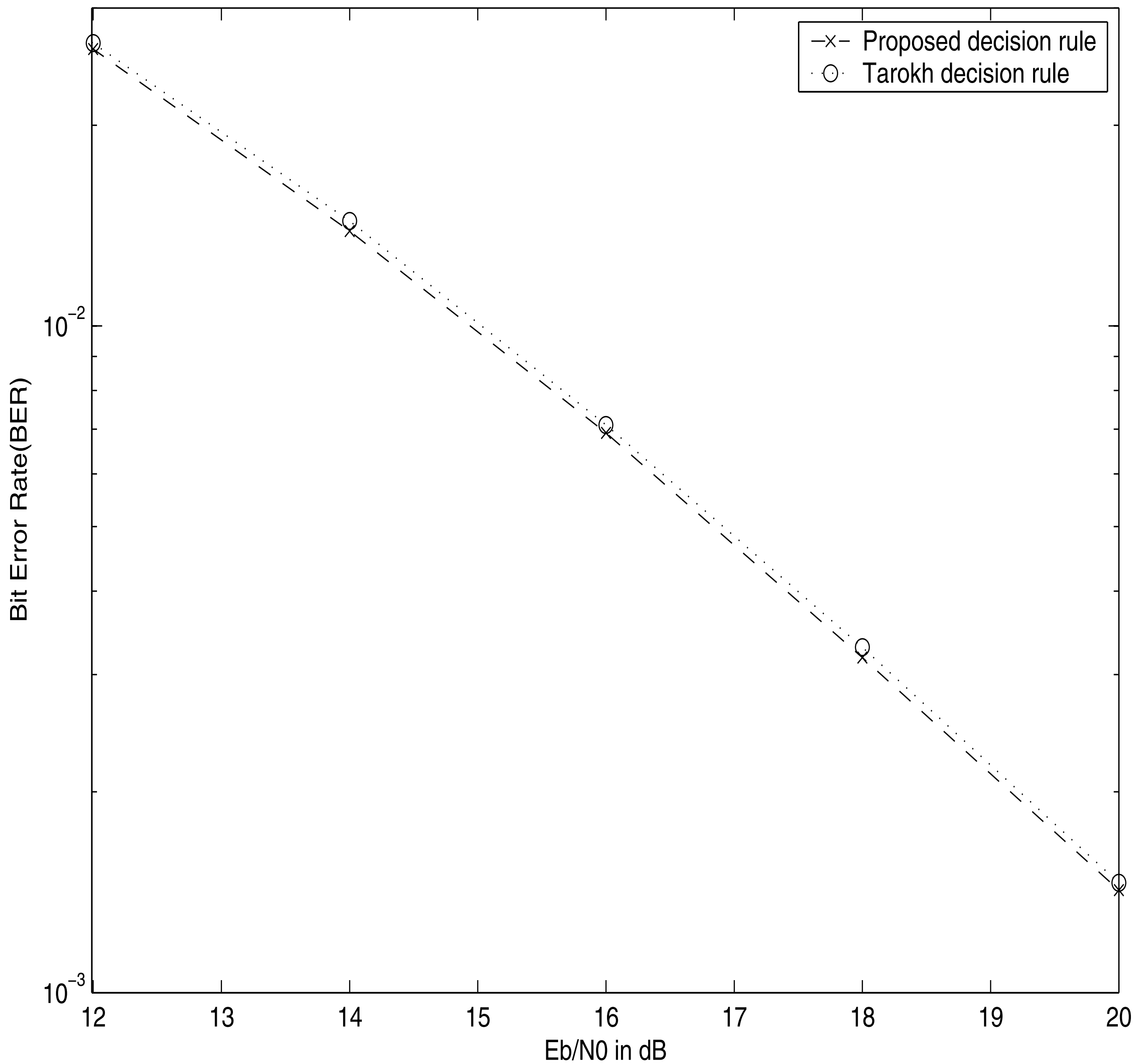

Fig. 4. BER performance for 16-QAM gray coded modulation (512 bits/frame) with the STB coded system.

$$p_{r,\hat{h}|s}(R,\hat{H}|S) = \frac{1}{\pi^4 |C_r| |C_{\hat{h}}| \left(1-|\mu|^2\right)^2} \exp\left\{ \frac{-1}{2^2\sigma_r^2\sigma_{\hat{h}}^2 \left(1-|\mu|^2\right)} \left(2\sigma_{\hat{h}}^2 r^H r + 2\sigma_r^2 \hat{h}^H \hat{h} - 2^2 \sigma_r \sigma_{\hat{h}} \Re\left[r^H C_\mu \hat{h}\right]\right)\right\} \quad (31)$$

implementation.

Appendix

In this appendix, we derive the exact pdf of the received signal conditioned on the estimated channel parameter and the transmitted symbol sequences.

The received signal samples are complex Gaussian distributed with $\mathbf{r} \sim N_c(\mu_r, C_r) \in \Im^2$, where $\Im^2$ denotes complex vector of dimension 2 and N^c denotes complex normal distribution. Here the mean of the distribution is μ_r, the covariance matrix is C_r. It is straightforward that $\mu_r = 0$, $C_r = 2\sigma_r^2 I_2$, where I_2 denotes 2×2 unit matrix. Hence the pdf of the complex received signal vector can be expressed as [9]

$$p_{r|s}(R|S) = \frac{1}{\pi^2 |C_r|} \exp\left(-r^H C_r^{-1} r\right). \quad (27)$$

Here s is the transmitted signal vector at any time slot and $|X|$ denotes the determinant, while X^H denotes Hermitian of the matrix X.

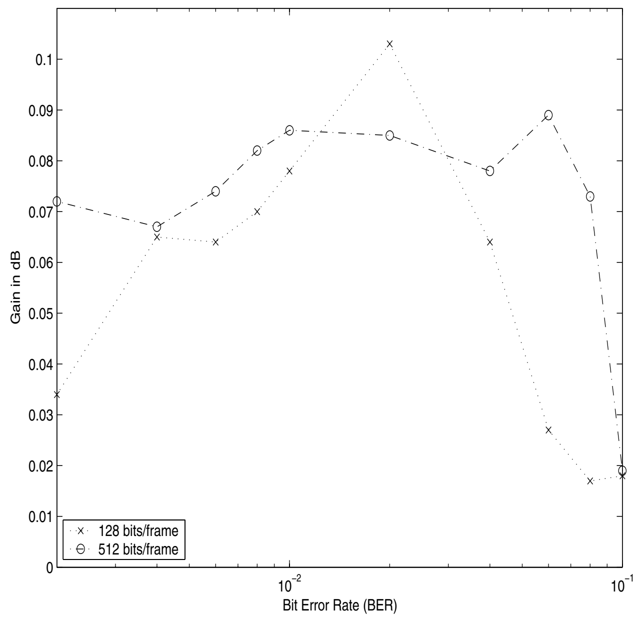

Fig. 5. Gain in dB obtained using the proposed metric over Tarokh's one for 16-QAM gray coded modulation.

Similarly distribution of the estimated channel parameter is $\hat{h} \sim N_c(\mu_{\hat{h}}, C_{\hat{h}}) \in \Im^2$, where the mean is $\mu_{\hat{h}}$ and the covariance matrix is $C_{\hat{h}}$. Again, we have $\mu_{\hat{h}} = 0$ and $C_{\hat{h}} = 2\sigma_{\hat{h}}^2 I_2$. Thus $\hat{\mathbf{h}}$ has the complex distribution function as follows

$$p_{\hat{h}}(\hat{H}) = \frac{1}{\pi^2 \left|C_{\hat{h}}\right|} \exp\left(-\hat{h}^H C_{\hat{h}}^{-1} \hat{h}\right). \tag{28}$$

Now, the joint distribution function of $\mathbf{r}$ and $\hat{\mathbf{h}}$ is [7]

$$\begin{aligned} & p_{r,\hat{h}|s}(R, \hat{H}|S) \\ & = \frac{1}{\pi^4 \left|C_{r,\hat{h}}\right|} \exp\left(-\begin{bmatrix} r \\ \hat{h} \end{bmatrix}^H C_{r,\hat{h}}^{-1} \begin{bmatrix} r \\ \hat{h} \end{bmatrix}\right) \end{aligned} \tag{29}$$

where the correlation matrix can be expressed as

$$\begin{aligned} C_{r,\hat{h}} &= E\left[\begin{bmatrix} r \\ \hat{h} \end{bmatrix} \begin{bmatrix} r^H & \hat{h}^H \end{bmatrix}\right] \\ &= \begin{bmatrix} 2\sigma_r^2 I_2 & 2\sigma_h^2 G \\ 2\sigma_h^2 G^H & 2\sigma_{\hat{h}}^2 I_2 \end{bmatrix}. \end{aligned} \tag{30}$$

We have found that $\left|C_{r,\hat{h}}\right| = 2^4 \sigma_r^4 \sigma_{\hat{h}}^4 \left(1 - |\mu|^2\right)^2$ and

$$C_{r,\hat{h}}^{-1} = \frac{1}{\left|C_{r,\hat{h}}\right|^{1/2}} \begin{bmatrix} 2\sigma_{\hat{h}}^2 I_2 & -2\sigma_r \sigma_{\hat{h}} C_\mu \\ -2\sigma_r \sigma_{\hat{h}} C_\mu^H & 2\sigma_r^2 I_2 \end{bmatrix} \tag{31}$$

where C_μ is defined as

$$C_\mu = \begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}$$

Putting all these values in (29) and doing simple multiplications, we reach (31), where $\Re[X]$ denotes the real axis portion of the complex scaler X, and the definition of $|\mu|^2$ is given in (7).

According to Bayes theorem, we can use the following expression to get the pdf of the received signal conditioned on the estimated channel parameters:

$$p_{r|\hat{h},s}(R|\hat{H}, S) = \frac{p_{r,\hat{h}|s}(R, \hat{H}|S)}{p_{\hat{h}|s}(\hat{H}|S)} = \frac{p_{r,\hat{h}|s}(R, \hat{H}|S)}{p_{\hat{h}}(\hat{H})} \tag{32}$$

as $\hat{h}$ is independent of the transmitted symbols.

Putting values from (28) and (31) in (32), and doing some simple manipulations, we finally reach the desired conditional pdf, which is given in (10). This is the exact conditional pdf required for derivation of the decision rule in the presence of partial knowledge of CSI.

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