A Low Complex Decoding Method for Space-Time Block Codes with Partial CSI

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Abstract. In this paper, we derive a low complex decoding metric for space-time block (STB) codes over quasi-static (slowly changing) flat fading channels with partial knowledge of channel state information (CSI). Here it is assumed that the channel fading coefficients are estimated by inserting pilot sequences in the transmitted signals. A channel estimator is assumed to provide us with an unbiased channel estimate with a certain error variance and the exact decoder metric is calculated for this case. By including the variance of the channel estimation error in the decoder metric derivation of the STB codes, we derive the exact probability density function (pdf) of the received signals conditioned on the estimated channel fading coefficients and transmitted symbol sequences. A modified decision rule is easily obtained from this pdf and then it is shown how the new metric can be applied to the most important cases discussed in the literature. Simulation results show that the proposed metric can significantly reduce the computational complexity without any performance degradation compared with the metric proposed by Tarokh.

1 Introduction

Simplicity of practical implementation and feasibility of having multiple antennas at the base station is the main reason to choose transmit diversity as a method of combating detrimental effects in the wireless communication system. While receive diversity like the maximal-ratio receiver combining (MRRC) scheme uses multiple receive antennas, the space-time block (STB) codes use multiple transit antennas to achieve performance gain. The STB codes can achieve diversity gain while maintaining small physical size of the receiver. Historically, this scheme was first proposed by Alamouti [1] for two transmitter antennas and multiple receiver antennas. Later, the scheme was generalized for any number of transmitter and receiver antennas [12–14, 17].

In [1], a decision rule is derived for decoding of the STB codes when the receiver has complete knowledge of the channel state information (CSI). In practice, however, the receiver never has the perfect knowledge of the CSI, as the channel parameters are random variables. The parameters are estimated using a channel estimation technique as the decision rule requires the knowledge of these parameters. Thus all practical communication systems can be considered as systems with imperfect channel knowledge. For such STB coded systems with imperfect channel knowledge, if we employ the decision rule for perfect knowledge of CSI using imperfect channel parameter estimates in place of actual channel parameters, performance degradation of the whole system is observed due to mismatch. Performance degradation due to this type of mismatch in the channel parameters in the decision rule has been addressed in the standard literature [16]. It is shown in [2] that a STB coded system is more sensitive to the channel estimation error than straightforward two branch diversity schemes, because of their dependency on the removal of the cross-terms in the decision rule. This dependency on the channel estimation error increases as the number of transmitter and receiver antenna increases to achieve high error performance [6].

To resolve this issue of mismatch due to the imperfect channel estimation, the case of partial knowledge of CSI was discussed and a complex decision rule was proposed by Tarokh [11, 13]. The partial knowledge of CSI utilized was the variance of the estimation error of the channel parameters, which can be easily and reliably obtained.

A systematic approach to include variance of the channel estimation error has been done by Frenger in [5] for Turbo coded systems. A similar approach is taken in this paper to calculate a new metric for a STB coded system proposed by Alamouti, which can easily be generalized for similar type of systems. Here a low complex decision rule is derived with no performance degradation compared with the metric proposed by Tarokh in [13]. Estimation error variance of the pilot symbol channel estimation technique is also investigated.

The organization of this paper is as follows. Section 2 describes the system model including a channel estimation technique. In Section 3, a new decoding metric for STB codes is derived from the exact pdf of the received signal conditioned on the estimated channel parameters. In Section 4, the proposed scheme is compared with the state-of-the-art scheme proposed by Tarokh. Numerical results are presented in Section 5, and conclusions are drawn in Section 6. Finally, the derivation of the exact conditional pdf of the received signal is given in the Appendix.

2 System Model

Here we consider a wireless communication system with n transmitter antennas at the base station and one receiver antenna at the remote station. Extension of formulations for m receiver antennas is straightforward. The encoder uses a generator matrix to encode the modulated data into different transmitter sequences, maintaining orthogonality of the sequences. Each of the n transmitter antennas simultaneously transmit one symbol $s_{i,t}, i = 0, 1, 2, \cdots, n-1$ at any time slot t. The received signals at the receiver are combined and then decoded using the maximum likelihood (ML) decoding algorithm to obtain the data bits.

We assume a flat fading wireless channel with the path gain defined to be h_i from the transmitter antenna i to the receiver antenna. The path gains are modeled as samples of zero mean, independent complex Gaussian random variables with the variance defined as $E[|h_i|^2] = 2\sigma_h^2$, where $E[x]$ denotes the expected value of x. Furthermore, the wireless channel is assumed to be quasi-static so that the path gains are constant over a frame of length L and vary from one frame to another. This incorporates the required assumption of the STB decoder to have constant fading for all the symbols of a block of transmitted symbol sequences having length of l. A frame consists of an integer number of blocks.

In this paper, we consider the STB coded system considered in [1] with 2 transmitter antennas and one receiver antenna. Data are encoded with STB encoder using the generator matrix

$$\mathbf{G} = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} \tag{1}$$

where each column represents the signals transmitted from a particular antenna at different time slots and each row represents the signal vector transmitted from all transmitter antennas at a particular time slot. Here x^* denotes the complex conjugate of x.

After sampling of the received signal using the matched filter at the receiver, we have samples of the received signals [15]. At the receiver antenna, the received signals over two consecutive symbol periods, denoted by r_0 and r_1 for time t and $t+T$, respectively, can be expressed as

$$\mathbf{r} = \mathbf{G}\mathbf{h} + \mathbf{n}, \tag{2}$$

where

$$\mathbf{r} = \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_0 \\ h_1 \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} n_0 \\ n_1 \end{bmatrix},$$

and n_0 and n_1 are independent complex Gaussian random variables with zero mean and variance $E[|n_0|^2] = E[|n_1|^2] = 2\sigma_n^2$. Actual fading parameter $\mathbf{h}$ is also a vector of complex valued Gaussian distributed elements with mean zero and variance $E[|h_0|^2] = E[|h_1|^2] = 2\sigma_h^2$. All real and imaginary parts of $\mathbf{h}$ and $\mathbf{n}$ are assumed to be independent. It is straightforward to show that the variance of the sampled zero mean received signal becomes

$$E[|r_0|^2] = E[|r_1|^2] = 2\sigma_r^2 = 2\sigma_h^2(|s_0|^2 + |s_1|^2) + 2\sigma_n^2.$$

Now, the estimated channel parameter model can be expressed as

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e}, \tag{3}$$

where the estimated channel vector $\hat{\mathbf{h}} = [\hat{h}_0 \ \hat{h}_1]^T$ and the error vector $\mathbf{e} = [e_0 \ e_1]^T$. Here $\mathbf{e}$ is complex valued Gaussian distributed estimation error with $E[|e_0|^2] = E[|e_1|^2] = 2\sigma_e^2$ and $E[\mathbf{e}] = 0$. Assuming $\mathbf{h}$ and $\mathbf{e}$ to be independent, we find that $\hat{\mathbf{h}}$ is also a vector of complex valued Gaussian distributed random variables with zero mean and variance defined as $E[|\hat{h}_0|^2] = E[|\hat{h}_1|^2] = 2\sigma_{\hat{h}}^2 = 2\sigma_h^2 + 2\sigma_e^2$. It is shown in [3] that this channel estimate model is valid for pilot-based channel estimation schemes. Furthermore, in [4], this model is shown to hold for decision-directed channel estimation schemes, assuming that the previous data symbols used for channel estimation were correctly detected. This model can be used in other channel estimation models as well [8].

In this paper, we use a pilot-based channel estimation technique, where the channel fading coefficients are estimated by inserting orthogonal pilot sequences in the transmitted signals. In this method, some pilot sequences are inserted at the beginning (or middle) of a data frame. The receiver has perfect knowledge of the positions and magnitudes of the pilot sequences. For multiple transmit antennas, the pilot sequence of any transmitter antenna must be orthogonal to other pilot sequences from other transmitter antennas to simplify channel estimator structure. For a system with n transmitter antennas, n different pilot sequences $\mathbf{P}_0, \mathbf{P}_1, ..., \mathbf{P}_{n-1}$ with the same length are needed. Let k be the length of the pilot sequences, i.e., $\mathbf{P}_i = [P_{i,0}\ P_{i,1}\ ...\ P_{i,k-1}]^T$ for the ith transmitter. To satisfy the orthogonality property, the pilot sequence of the ith transmitter has to satisfy the condition

$$\langle \mathbf{P}_i, \mathbf{P}_j \rangle \triangleq \mathbf{P}_i^H \mathbf{P}_j = \begin{cases} \|\mathbf{P}_i\|^2 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

where $\langle \mathbf{P}_i, \mathbf{P}_j \rangle$ denotes the inner product of $\mathbf{P}_i$ and $\mathbf{P}_j$ and the superscript H denotes the complex conjugate transpose (i.e., the Hermitian transpose).

The receiver isolates the received signals due to the pilot symbols and sends those to the channel estimator for initial estimation of channel before decoding of the received signals due to data symbols. During the channel estimation, the received signal at the receiver antenna at time t can be represented by

$$r_t = \sum_{i=1}^{n} h_i \cdot P_{i,t} + n_t. \tag{4}$$

The received signal and noise sequence at the antenna can be represented as $\mathbf{r}_p = [r_0\ r_1\ ...\ r_{k-1}]^T$, and $\mathbf{n}_p = [n_0\ n_1\ ...\ n_{k-1}]^T$, respectively. The receiver estimates the channel fading parameters h_i by using the observed sequence $\mathbf{r}_p$. Since the pilot sequences $P_0, P_1, ..., P_{n-1}$ have been designed to be orthogonal to each other, the minimum mean square error (MMSE) estimate of h_i is given by [13]

$$\begin{aligned} \hat{h}_i &= \frac{\langle \mathbf{P}_i, \mathbf{r}_p \rangle}{\langle \mathbf{P}_i, \mathbf{P}_i \rangle} = \frac{\mathbf{P}_i^H \mathbf{r}_p}{\|\mathbf{P}_i\|^2} = \frac{\mathbf{P}_i^H (h_i \mathbf{P}_i + \sum\limits_{j=1, j\neq i}^{n} h_j \mathbf{P}_j + \mathbf{n}_p)}{\|\mathbf{P}_i\|^2}, \\ &= h_i + e_i, \end{aligned} \tag{5}$$

where e_i is the estimation error due to the noise, given by $e_i = \langle \mathbf{P}_i, \mathbf{n}_p \rangle / \langle \mathbf{P}_i, \mathbf{P}_i \rangle$. Since n_p is a zero-mean complex Gaussian random variable with single-sided power spectral density N_0, the estimation error e_i has a zero mean and single-sided power spectral density N_0/k [13].

Here in order to derive a new decision metric, we obtain the exact pdf of the received signal conditioned on the estimated channel parameters and transmitted symbol sequences. To simplify our calculations, the following cross correlation coefficients are defined:

$$\begin{aligned} \mu_{00} &\triangleq \frac{E[r_0 h_0^*]}{\sqrt{\mathrm{var}(r_0)\mathrm{var}(h_0)}} \qquad \mu_{01} \triangleq \frac{E[r_0 h_1^*]}{\sqrt{\mathrm{var}(r_0)\mathrm{var}(h_1)}} \\ \mu_{10} &\triangleq \frac{E[r_1 h_0^*]}{\sqrt{\mathrm{var}(r_1)\mathrm{var}(h_0)}} \qquad \mu_{11} \triangleq \frac{E[r_1 h_1^*]}{\sqrt{\mathrm{var}(r_1)\mathrm{var}(h_1)}}. \end{aligned}$$

It can easily be shown that $\mu_{00} = \mu_{11}^* = s_0\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$ and $\mu_{01} = -\mu_{10}^* = s_1\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$. We further define

$$|\mu|^2 \triangleq |\mu_{ij}|^2 + |\mu_{mn}|^2 = (|s_0|^2 + |s_1|^2)\frac{\sigma_h^4}{\sigma_r^2\sigma_{\hat{h}}^2}, \tag{6}$$

where $i, j, m, n \in \{0, 1\}$ with the condition that if $i = m$, then $j \neq n$ or vice versa.

As shown in the Appendix, the required pdf can be expressed as follows:

$$p_{\mathbf{r}|\hat{\mathbf{h}},\mathbf{s}}(\mathbf{R}|\hat{\mathbf{H}},\mathbf{s}) = \frac{1}{(2\pi)^2\sigma_r^4\left(1-|\mu|^2\right)^2}\exp\left[\frac{-1}{2\sigma_r^2\left(1-|\mu|^2\right)}\right.$$
$$\times\left.\left(\left|R_0 - \left(\mu_{00}\hat{H}_0 + \mu_{01}\hat{H}_1\right)\frac{\sigma_r}{\sigma_h}\right|^2 + \left|R_1 - \left(\mu_{00}^*\hat{H}_1 - \mu_{01}^*\hat{H}_0\right)\frac{\sigma_r}{\sigma_h}\right|^2\right)\right], \tag{7}$$

where $\mathbf{s} = [s_0, s_1]^T$ is the vector of signals transmitted at a particular time slot.

3 Derivation of a New Decision Metric

We can easily find that the conditional pdf derived in (7) can be rewritten as the product of conditional pdf's $p_{r_0|\hat{\mathbf{h}},\mathbf{s}}(R_0|\hat{\mathbf{H}},\mathbf{s})$ and $p_{r_1|\hat{\mathbf{h}},\mathbf{s}}(R_1|\hat{\mathbf{H}},\mathbf{s})$,

$$p_{\mathbf{r}|\hat{\mathbf{h}},\mathbf{s}}(\mathbf{R}|\hat{\mathbf{H}},\mathbf{s}) = \frac{1}{2\pi\sigma_r^2\left(1-|\mu|^2\right)}\exp\left[\frac{-\left|R_0 - \left(\mu_{00}\hat{H}_0 + \mu_{01}\hat{H}_1\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2}{2\sigma_r^2\left(1-|\mu|^2\right)}\right]$$
$$\times\frac{1}{2\pi\sigma_r^2\left(1-|\mu|^2\right)}\exp\left[\frac{-\left|R_1 - \left(\mu_{00}^*\hat{H}_1 - \mu_{01}^*\hat{H}_0\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2}{2\sigma_r^2\left(1-|\mu|^2\right)}\right], \tag{8}$$

or

$$p_{\mathbf{r}|\hat{\mathbf{h}},\mathbf{s}}(\mathbf{R}|\hat{\mathbf{H}},\mathbf{s}) = p_{r_0|\hat{\mathbf{h}},\mathbf{s}}(R_0|\hat{\mathbf{H}},\mathbf{s})p_{r_1|\hat{\mathbf{h}},\mathbf{s}}(R_1|\hat{\mathbf{H}},\mathbf{s}). \tag{9}$$

Thus, given $\hat{\mathbf{H}}$ and $\mathbf{s}$, the random variables r_0 and r_1 are independent, complex-valued Gaussian distributed with conditional means

$$E\left[r_0|\hat{\mathbf{H}},\mathbf{s}\right] = (\mu_{00}\hat{H}_0 + \mu_{01}\hat{H}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}, \tag{10}$$

$$E\left[r_1|\hat{\mathbf{H}},\mathbf{s}\right] = (\mu_{00}^*\hat{H}_1 - \mu_{01}^*\hat{H}_0)\frac{\sigma_r}{\sigma_{\hat{h}}}, \tag{11}$$

respectively, and equal second moment

$$E\left[\left|r_0|\hat{\mathbf{H}},\mathbf{s}\right|^2\right] = E\left[\left|r_1|\hat{\mathbf{H}},\mathbf{s}\right|^2\right] = 2\sigma_r^2(1-|\mu|^2). \tag{12}$$

Assuming that all the signals in the modulation constellation are equiprobable, a ML decoder chooses a pair of signals from the signal modulation constellation to minimize the distance metric

$$d^2(R_0, E[r_0|\hat{\mathbf{H}}, \mathbf{s}]) + d^2(R_1, E[r_1|\hat{\mathbf{H}}, \mathbf{s}]) \tag{13}$$

over all possible values of $\hat{s}$, the detected signal sequence vector. Here $d^2(x, y)$ is the squared Euclidean distance between signals x and y calculated using the expression $d^2(x, y) = (x - y)(x^* - y^*) = |x - y|^2$.

Substituting (10) and (11) into (13) leads us to the minimization problem of the following distance metric

$$\left|R_0 - (\mu_{00}\hat{H}_0 + \mu_{01}\hat{H}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2 + \left|R_1 - (\mu_{00}^*\hat{H}_1 - \mu_{01}^*\hat{H}_0)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2 \tag{14}$$

for all transmitted symbol sequences.

After expanding the above metric and deleting the terms independent of the transmitted symbols, we reach the following equivalent metric to be minimized

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\Big(-R_0\hat{H}_0^* s_0^* - R_0^*\hat{H}_0 s_0 - R_0\hat{H}_1^* s_1^* - R_0^*\hat{H}_1 s_1 - R_1\hat{H}_1^* s_0 - R_1^*\hat{H}_1 s_0^* + R_1\hat{H}_0^* s_1$$
$$+R_1^*\hat{H}_0 s_1^*\Big) + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|s_0|^2|\hat{H}_0|^2 + |s_1|^2|\hat{H}_1|^2 + |s_0|^2|\hat{H}_1|^2 + |s_1|^2|\hat{H}_0|^2\right).$$

We can decompose this term into two parts for the sake of the simplicity of the detection process, as

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left[-(R_0\hat{H}_0^* + R_1^*\hat{H}_1)s_0^* - (R_0^*\hat{H}_0 + R_1\hat{H}_1^*)s_0\right] + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{H}_0|^2 + |\hat{H}_1|^2\right)|s_0|^2,$$

which is only a function of s_0, and

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left[-(R_0\hat{H}_1^* - R_1^*\hat{H}_0)s_1^* - (R_0^*\hat{H}_1 - R_1\hat{H}_0^*)s_1\right] + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{H}_0|^2 + |\hat{H}_1|^2\right)|s_1|^2,$$

which is only a function of s_1. Thus the minimization problem given in (14) is reduced to minimizing these two parts separately. This leads us to a faster decoding process with less complexity, especially for higher order modulation schemes. After some rearrangement and manipulation of the above two expressions, we can obtain the maximum likelihood estimates of s_0 and s_1, denoted by $\hat{s}_0$ and $\hat{s}_1$, respectively, as follows:

$$\hat{s}_0 = \arg\min_{s_0\in\Omega_d}\left[\left|(R_0\hat{H}_0^* + R_1^*\hat{H}_1)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_0\right|^2 + \eta_o\left|s_0\right|^2\right], \tag{15}$$

$$\hat{s}_1 = \arg\min_{s_1\in\Omega_d}\left[\left|(R_0\hat{H}_1^* - R_1^*\hat{H}_0)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1\right|^2 + \eta_o\left|s_1\right|^2\right], \tag{16}$$

where $\eta_o = \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{H}_0|^2 + |\hat{H}_1|^2\right) - 1$, and Ω_d is the discrete set of admissible symbols.

4 Some important special cases

The general decision rules (15) and (16) are the main contributions of this paper. Using these decision rules, the following important special cases can be explained:

- *Ideal case*: In this case, originally developed in [1], the channel parameters are assumed to be known at the receiver. Based on this assumption, (15) and (16) can be simplified as follows:

$$\hat{s}_0 = \arg\min_{s_0 \in \Omega_d} \Big[|(R_0 H_0^* + R_1^* H_1) - s_0|^2 + \eta |s_0|^2 \Big], \tag{17}$$

$$\hat{s}_1 = \arg\min_{s_1 \in \Omega_d} \Big[|(R_0 H_1^* - R_1^* H_0) - s_1|^2 + \eta |s_1|^2 \Big], \tag{18}$$

  where $\eta = |H_0|^2 + |H_1|^2 - 1$.

- *Alamouti's Scheme* [1]: This scheme, proposed by Alamouti in [1], is based on an initial assumption that the channel parameters have been estimated perfectly at the receiver. For perfect channel estimates, $\sigma_{\hat{h}}^2 = \sigma_h^2$, and $\hat{H}_i = H_i$, where $i \in \{0, 1\}$. Then (15) and (16) can be simplified to

$$\hat{s}_0 = \arg\min_{s_0 \in \Omega_d} \Big[|(R_0 \hat{H}_0^* + R_1^* \hat{H}_1) - s_0|^2 + \hat{\eta} |s_0|^2 \Big], \tag{19}$$

$$\hat{s}_1 = \arg\min_{s_1 \in \Omega_d} \Big[|(R_0 \hat{H}_1^* - R_1^* \hat{H}_0) - s_1|^2 + \hat{\eta} |s_1|^2 \Big], \tag{20}$$

  where $\hat{\eta} = |\hat{H}_0|^2 + |\hat{H}_1|^2 - 1$.

  On the other hand, this scheme can be obtained simply by taking (17) and (18) for known parameters and replacing the known channel parameters with their estimates. In other words, the estimated parameters are treated as if they were the true channel parameters; i.e., the uncertainty in the parameter estimates are not considered. This approach is commonly called *certainty equivalence*, [18]. Since the scheme proposed by Alamouti ignores any uncertainty associated with the channel parameters, the solutions obtained in (19) and (20), are not optimal but they can be considered as approximate or suboptimal solutions.

- *Tarokh's scheme* [13]: We now compare the proposed metric with the state-of-the-art metric given by Tarokh [11, 13] in the presence of the channel estimation error. Originally derived for space-time trellis codes, the metric is however generally accepted for STB codes. The mean and variance of the distribution function of the random variable r_t conditioned on $\hat{h}_i$ as given by Tarokh are $\mu_{r'} = \mu'/(\sqrt{2}\sigma_{\hat{h}})\sqrt{E_s}\sum_{i=1}^{n} \hat{s}_{i,t}\hat{h}_i$ and $\sigma_{r'}^2 = N_0 + (1 - |\mu'|^2) E_s \sum_{i=1}^{n} \left|\hat{s}_{i,t}^2\right|$, respectively. Here $\mu' = 1/\sqrt{1 + 2\sigma_e^2}$ is the correlation coefficient, $N_0 = 2\sigma_n^2$ is the noise variance and E_s is the energy per symbol, which is the factor by which the elements of the signal constellation are contracted to make the average energy of the constellation as 1. The decision metric proposed by Tarokh for n transmitter antennas and one receiver antenna can be written as

$$\sum_{t=1}^{l}\left[\frac{\left|r_t-\frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2}{N_0+(1-|\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2}+\ln\Big(N_0+(1-|\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2\Big)\right]. \tag{21}$$

For any PSK (phase shift keying) constellation, the metric given in (21) is simplified to

$$\sum_{t=1}^{l}\left|r_t-\frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2. \tag{22}$$

For the case of normalized Rayleigh fading channel, which is assumed by Tarokh, we have $\sigma_h^2 = 0.5$ and $\sqrt{E_s} = 1$. Considering the case of 2 transmitter and 1 receiver antenna, it can easily be shown that (22), the metric proposed by Tarokh in [13], is identical with (14), the metric derived in this paper.

To address the complexity issues and memory requirements of the proposed scheme compared with that of Tarokh's scheme, some assumptions are made in order to simplify our comparative study. In this simplified approach, we are interested only in the complexity of the decision rule as the difference of the two schemes lies in the decision rules. Since multiplications and divisions are the most complex basic operations in designing the signal processors, we base our calculations on the requirements of these operations in the decision rules. We assume the complexity of a multiplier, a divisor, a squaring or a logarithm operation be the same. The complexity index is calculated by simply adding the required number of multiplications, divisions, squaring and logarithm operations. Table I gives the comparison of the proposed scheme with Tarokh's scheme for p-QAM and p-PSK modulation, where p is the number of states of the modulation scheme. The metrics are to be computed for each possible combination of the signals transmitted for each state of the modulation scheme. Assume that factors are to be calculated once and then stored for reuse in the later computations, rather than calculating each time. This results in minimized processing time by reducing the number of multiplications with increasing storage size. The metrics proposed by Tarokh have to be computed p^2 times for different combinations of $\hat{s}_0$ and $\hat{s}_1$ and compared with each other. However, in the proposed scheme, each metric has to be computed only p times because of the variable separation operation during the derivation of the metrics. This is why the complexity of the decision rule of the proposed scheme is much lower than that of Tarokh's scheme.

5 Simulation Results

The simulations have been performed using Alamouti's scheme for STB codes with two transmitter and one receiver antenna. The channel is assumed to be

Modulation Scheme	Decision Metric	Complexity Index	Memory Requirements	Number of Comparisons
p-QAM	Proposed (15)(16)	$17\times p$	10	$p(p-1)$
p-QAM	Tarokh(21)	$20\times p^2$	13	$p^2(p^2-1)/2$
p-PSK	Proposed (15)(16)	$17\times p$	10	$p(p-1)$
p-PSK	Tarokh(22)	$11\times p^2$	11	$p^2(p^2-1)/2$

Table 1: Comparison of the Complexity Issues of the Proposed Scheme with Tarokh's Scheme.

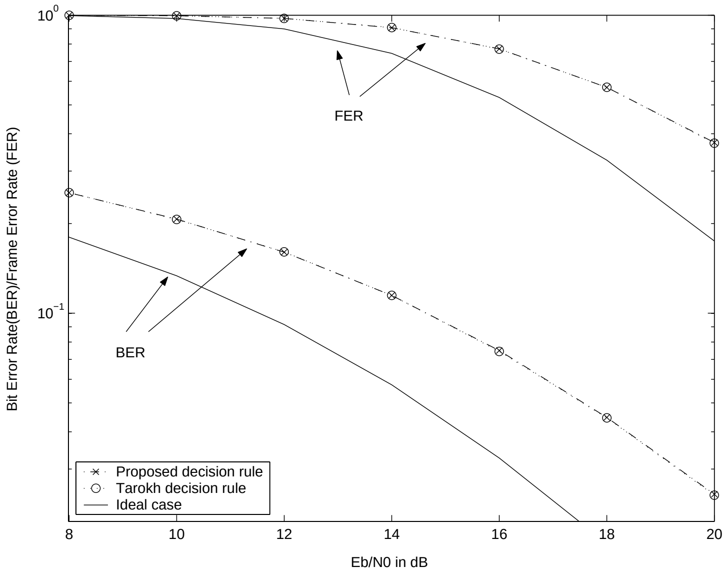

Figure 1: BER and FER performance for 16-PSK modulation (128 bits/frame) with a STB coded system using two pilot symbols per frame for the proposed decision rule, Tarokh's decision rule and the ideal case with known CSI.

quasi-static and flat faded. Normalized Rayleigh fading is assumed with the variance per complex dimension as $\sigma_h^2 = 0.5$. For pilot symbol insertion, the total transmit energy per frame is kept constant for a fair comparison. The transmitted signal sequences are modulated using a 16-PSK.

Fig. 1 shows the bit error rate (BER) and frame error rate (FER) performance curves of the system under consideration with 16-PSK modulation scheme with the proposed decision rule and the decision rule proposed by Tarokh. The case with known CSI is given for comparison purpose which acts as a lower bound. For the channel estimation, two pilot symbols are added to each frame containing information of 128 bits, which is an overhead of 1/16. As seen, the proposed metric and the metric proposed by Tarokh have the same performance in terms of BER and FER; however, the proposed metric has lower complexity as shown in Table I.

6 Conclusion

In this paper, a low complex decoding metric for space-time block (STB) codes over quasi-static (slowly changing) flat fading channels in the presence of channel estimation errors is derived. Here it is assumed that the channel fading coefficients are estimated by inserting pilot sequences in the transmitted signals. A channel estimator is assumed to provide us with an unbiased channel estimate with a certain error variance and the exact decoder metric is calculated for this case. By including the variance of the channel estimation error in the decoder metric derivation of the STB codes, we derive the exact probability density function (pdf) of the received signals conditioned on the estimated channel fading coefficients and transmitted symbol sequences. A modified decision rule is easily obtained from this pdf and then it is shown how the new metric can be applied to the most important cases discussed in the literature. Simulation results show that the proposed metric can significantly reduce the computational complexity with no performance degradation compared with the metric proposed by Tarokh.

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Appendix:

In this appendix, we derive the exact pdf of the received signal conditioned on the estimated channel parameter and the transmitted symbol sequences.

The received signal vector $\mathbf{r}$ is complex Gaussian distributed with mean μ_r and covariance matrix C_r. It is straightforward to show that $\mu_r = 0$, $C_r = 2\sigma_r^2 I_2$, where I_2 denotes a 2×2 identity matrix. Hence the pdf of the complex received signal vector can be expressed as [9]

$$p_{r|\mathbf{s}}(R|\mathbf{s}) = \frac{1}{\pi^2 |C_r|} \exp\left(-r^H C_r^{-1} r\right). \tag{23}$$

Here $\mathbf{s}$ is the transmitted signal vector at any time slot and $|X|$ denotes the determinant, while X^H denotes Hermitian of the matrix X. Similarly distribution of the estimated channel parameter vector $\hat{h}$ is complex Gaussian with mean $\mu_{\hat{h}}$

and covariance matrix $C_{\hat{h}}$. Again, we have $\mu_{\hat{h}} = 0$ and $C_{\hat{h}} = 2\sigma_{\hat{h}}^2 I_2$. Thus $\hat{\mathbf{h}}$ has the complex distribution function as follows:

$$p_{\hat{h}}(\hat{H}) = \frac{1}{\pi^2 \left|C_{\hat{h}}\right|} \exp\left(-\hat{h}^H C_{\hat{h}}^{-1} \hat{h}\right). \tag{24}$$

Now, we obtain the joint distribution function of $\mathbf{r}$ and $\hat{\mathbf{h}}$ conditiond on $\mathbf{s}$ as [7]

$$p_{r,\hat{h}|\mathbf{s}}(R, \hat{H}|\mathbf{s}) = \frac{1}{\pi^4 \left|C_{r,\hat{h}}\right|} \exp\left(-\begin{bmatrix} r \\ \hat{h} \end{bmatrix}^H C_{r,\hat{h}}^{-1} \begin{bmatrix} r \\ \hat{h} \end{bmatrix}\right), \tag{25}$$

where the correlation matrix can be expressed as

$$C_{r,\hat{h}} = E\left\{\begin{bmatrix} r \\ \hat{h} \end{bmatrix} \begin{bmatrix} r^H & \hat{h}^H \end{bmatrix}\right\} = \begin{bmatrix} 2\sigma_r^2 I_2 & 2\sigma_h^2 G \\ 2\sigma_h^2 G^H & 2\sigma_{\hat{h}}^2 I_2 \end{bmatrix}.$$

We have found that $\left|C_{r,\hat{h}}\right| = 2^4 \sigma_r^4 \sigma_{\hat{h}}^4 \left(1 - |\mu|^2\right)^2$ and

$$C_{r,\hat{h}}^{-1} = \frac{1}{\left|C_{r,\hat{h}}\right|^{1/2}} \begin{bmatrix} 2\sigma_{\hat{h}}^2 I_2 & -2\sigma_r \sigma_{\hat{h}} C_\mu \\ -2\sigma_r \sigma_{\hat{h}} C_\mu^H & 2\sigma_r^2 I_2 \end{bmatrix} \tag{26}$$

where C_μ is defined as

$$C_\mu \triangleq \begin{bmatrix} \mu_{00} & \mu_{01} \\ \mu_{10} & \mu_{11} \end{bmatrix}.$$

Putting all these values in (25) and doing simple multiplications, we obtain

$$p_{r,\hat{h}|\mathbf{s}}(R, \hat{H}|\ \mathbf{s}) = \frac{1}{\pi^4 \left|C_r\right| \left|C_{\hat{h}}\right| \left(1 - |\mu|^2\right)^2}$$
$$\times \exp\left\{\frac{-1}{2^2 \sigma_r^2 \sigma_{\hat{h}}^2 \left(1 - |\mu|^2\right)} \left(2\sigma_{\hat{h}}^2 r^H r + 2\sigma_r^2 \hat{h}^H \hat{h} - 2^2 \sigma_r \sigma_{\hat{h}} \Re\left[r^H C_\mu \hat{h}\right]\right)\right\}, \tag{27}$$

where $\Re[X]$ denotes the real axis portion of the complex scaler X, and the definition of $|\mu|^2$ is given in (6).

From Bayes' rule, the pdf of the received signal conditioned on the estimated channel parameters can be obtained as follows:

$$p_{r|\hat{h},\mathbf{s}}(R|\hat{H}, \mathbf{s}) = \frac{p_{r,\hat{h}|\mathbf{s}}(R, \hat{H}|\mathbf{s})}{p_{\hat{h}|\mathbf{s}}(\hat{H}|S)} = \frac{p_{r,\hat{h}|\mathbf{s}}(R, \hat{H}|\mathbf{s})}{p_{\hat{h}}(\hat{H})} \tag{28}$$

as $\hat{h}$ is independent of the transmitted symbols.

Substituting (24) and (27) into (28), and doing some simple manipulations leads to the desired conditional pdf, which is given in (7) or (8).