C(α)-type LM Test of Over-Identifying Moment Conditions in Time Series GEL Model

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Data Science Cluster Seminar
February 26, 2020

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Motivation for GEL Estimation: Consumption Based Capital Asset Pricing Model (CCAPM)

- Intertemporal optimization
  - $\underset{\{c_{t+i}\}}{Max}\ E[\sum_{i=0}^{\infty} \delta_0^i U(c_{t+i})|\mathcal{I}_t]$ subject to
  - $c_t + \sum_{j=1}^{N} p_{j,t} q_{j,t} = w_t + \sum_{j=1}^{N} r_{j,t} q_{j,t-m_j}$: budget constraint
  - $0 \leq \delta_0 \leq 1$: discount factor; $\mathcal{I}_t$: info set
- F.O.C. is given by the Euler equation
  - $p_{j,t} U'(c_t) = E[\delta_0^{m_j} r_{j,t+m_j} U'(c_{t+m_j})|\mathcal{I}_t]$

$\Rightarrow$ value of utility lost by foregoing consumption in period t to purchase 1 unit of asset j	=	E(discounted value of utility gained from consuming the return on investment in period $t + m_j$)

Motivation cont'd

- CRRA: $U(c_t) = \frac{c_t^{1-\gamma_0}}{1-\gamma_0}$ (γ_0: risk aversion parameter)
  - $E[\delta_0^{m_j}(\frac{r_{j,t+m_j}}{p_{j,t}})(\frac{c_{t+m_j}}{c_t})^{-\gamma_0}|\mathcal{I}_t] - 1 = 0$
  - 2 parameters to estimate: $\theta_0 \equiv (\delta_0, \gamma_0)$
  - Let $g(\cdot, \theta_0) = \delta_0^{m_j}(\frac{r_{j,t+m_j}}{p_{j,t}})(\frac{c_{t+m_j}}{c_t})^{-\gamma_0} - 1$
  - By iterated conditional expectation

    $E[g(\cdot, \theta_0)z_t] = E[[Eg(\cdot, \theta_0)|\mathcal{I}_t]z_t]] = 0$ for any $z_t \in \mathcal{I}_t$
  - E.g., $z_t = \frac{r_{j,t}}{p_{j,t-m_j}}, \frac{c_t}{c_{t-m_j}}$ for $j = 1, 2, \cdots, N$;

    and any macro/finance variable $\in \mathcal{I}_t$

Motivation cont'd

- $j = 1$ & $m_j = 1 \Rightarrow g(\cdot, \theta_0) = \delta_0(\frac{r_{t+1}}{p_t})(\frac{c_{t+1}}{c_t})^{-\gamma_0} - 1$

  $\Rightarrow \theta_0 \equiv (\delta_0, \gamma_0) \Rightarrow$ 2 parameters

- $z_t = (1, \frac{c_t}{c_{t-1}}, \frac{r_t}{p_{t-1}})' \Rightarrow$ 3 moment conditions

  $$E\Big[\Big(\delta_0(\frac{r_{t+1}}{p_t})(\frac{c_{t+1}}{c_t})^{-\gamma_0} - 1\Big)\Big(1\Big)\Big] = 0$$

  $$E\Big[\Big(\delta_0(\frac{r_{t+1}}{p_t})(\frac{c_{t+1}}{c_t})^{-\gamma_0} - 1\Big)\Big(\frac{c_t}{c_{t-1}}\Big)\Big] = 0$$

  $$E\Big[\Big(\delta_0(\frac{r_{t+1}}{p_t})(\frac{c_{t+1}}{c_t})^{-\gamma_0} - 1\Big)\Big(\frac{r_t}{p_{t-1}}\Big)\Big] = 0$$

  $\Rightarrow$ no. of moments, $q = 3 >$ no. of parameters, $p = 2$

  $\Rightarrow$ model over-identified

Estimation of Moment Condition Model: GMM & GEL

- $q < p \Rightarrow \theta_0$ under- or un-ID; not consistently estimable
- $q \geq p$ & $Rank\left[E\left(\frac{\partial g(\cdot,\theta_0)}{\partial \theta'}\right)\right] = p \Rightarrow \theta_0$ consistently estimable
  - $q = p \Rightarrow$ Just (Exact) ID $\rightarrow$ MOM: sample moments $= 0$

    i.e., $g_T(\hat{\theta}) = \frac{1}{T}\sum_{t=1}^{T} g_t(\hat{\theta}) = 0$
  - $q > p \Rightarrow$ Over-ID $\Rightarrow g_T(\hat{\theta}) \neq 0 \rightarrow$ GMM: minimize generalized distance of sample moments from zero

    i.e., $\hat{\theta} = \arg\min_{\theta} g_T(\theta)' W_T g_T(\theta)$

    W_T: symmetric PD weighting matrix (distance metric)
- Limitation of GMM: Biased estimators in finite samples
- One of potential sources of bias: Estimated optimal W_T

Estimation of Mom Cond Model: GMM & GEL cont'd

- Question: Could there be an alternative to GMM
  - asymptotically equivalent to GMM, but
  - does not require W_T?
- Answer: Yes, GEL

Existing Tests of Over-ID & Objective of My Paper

- Existing tests [Smith (2011, Econometric Theory)]:
  - $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ tests of over-id moment conditions in TS GEL model
  - Have certain disadvantages
- Objective of my paper:
  - Develop $C(\alpha)$-type LM ($\mathcal{LM}_\lambda$) test of over-id moment conditions in TS GEL model
  - Thereby counter such disadvantages

Motivation for C(α)-type LM ($\mathcal{LM}_\lambda$) Test

- Disadvantages of $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ tests: They require estimation of GEL parameters, which
  - can be relatively difficult in nonlinear models
  - requires solution of a saddle-point problem
  - grows in complexity as p and/or q grow
  - therefore, computationally more involved
  - and hence, less appealing
- C(α)-type LM ($\mathcal{LM}_\lambda$) test: Attractive alternative when model is nonlinear
  - but $\sqrt{T}$-consistent estimators are relatively easily available
  - e.g., efficient 2SGMM or CUGMM, and one can, therefore,
  - take advantage of GEL implied probabilities w/o computing GEL estimators
  - therefore, computationally less involved
  - and hence, more appealing

Contributions of C(α)-type LM ($\mathcal{LM}_\lambda$) Test Developed in this Paper

- Contributions of $\mathcal{LM}_\lambda$ test
  - an attractive alternative and addition to the existing repertoire of Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ tests of over-id moment conditions in TS GEL model
  - possesses superior/competitive size property relative to that of Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ tests as evidenced by the Monte Carlo study completed thus far

Brief Mention of My 3 C(α)-type LM ($\mathcal{LM}_\alpha, \mathcal{LM}_\alpha^w$, and $\mathcal{LM}_{\alpha\beta}^w$) Tests of Sub-Vector of Parameters under Strong & Weak ID in Time Series GEL Model Developed in 3 other Papers

The 3 tests of sub-vector of parameters under S & W ID developed in my 3 other papers are :

- *(i)* $\mathcal{LM}_\alpha$ test: C(α)-type LM test of sub-vector α when both the hypothesized sub-vector α and the nuisance parameter vector β are S ID

- *(ii)* $\mathcal{LM}_\alpha^w$ test: C(α)-type LM test of sub-vector α when the hypothesized sub-vector α contains both W & S ID parameters while the nuisance parameter vector β is S ID

- *(iii)* $\mathcal{LM}_{\alpha\beta}^w$ test: C(α)-type LM test of sub-vector α when both the hypothesized sub-vector α and the nuisance parameter vector β contain W & S ID parameters

Brief Overview of Time Series GEL Estimation

Let

- $\{z_t\}_{t=1}^{\infty}$: $l \times 1$ stationary strong mixing process
- $\{z_t\}_{t=1}^{T}$: $l \times 1$ random sample
- θ: $p \times 1$ unknown parameters
- $g(z_t, \theta) \equiv g_t(\theta)$: $q \times 1$ mom ind vectors, Borel funs
- $q \geq p$
- $E[g(z_t, \theta_0)] = 0$ (θ_0 unique) $\Rightarrow$ S ID of θ_0

Note: Dependence structure in TS data $\rightarrow$

- $Corr\left[g(z_t, \theta_0), \frac{\partial g(z_t, \theta_0)}{\partial \theta'}\right]$: one source of bias in estimators
- Question: How to remove this source of bias?

Brief Overview of Time Series GEL Estimation cont'd

Answer: Use kernel-smoothed $g_t(\theta)$

- $g_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} \kappa(\frac{s}{S_T}) g_{t-s}(\theta)$
  - $\kappa(\cdot)$: kernel fun
  - S_T: bandwidth

- Anatolyev [(2005), Econometrica]: shows
  - even when the moment indicator vectors are serially uncorrelated
  - but not IID across time

smoothing the moment indicator vectors using kernel functions can reduce the bias

Brief Overview of Time Series GEL Estimation cont'd

To estimate θ, construct GEL criterion fun:

- $\hat{P}_\rho(\theta, \lambda) = \sum_{t=1}^{T}[\rho(\kappa\lambda' g_{tT}(\theta)) - \rho(0)]/T$
  - $\rho(v) : \mathcal{V} \to \Re$; $\mathcal{V}$: open interval in $\Re$ containing zero
    - $\rho(v)$: scalar fun, concave, C^2 in $\mathcal{N}_0$
    - $\partial\rho(0)/\partial v = \partial^2\rho(0)/\partial v^2 =$ -1
  - $\kappa = \kappa_1/\kappa_2$; $\kappa_j = \int_{-\infty}^{\infty} \kappa(a)^j da$, $j = 1, 2$
  - $\Lambda_T(\theta) = \{\lambda \in \Re^q : \lambda' g_{tT}(\theta) \in \mathcal{V}\}$: Bounds on λ
- 3 common choices of $\rho(v)$:
  - $\rho(v) = -(1+v)^2/2$, $\mathcal{V} = \Re$: CU-GMM or EEL
  - $\rho(v) = ln(1-v)$, $\mathcal{V} = (-\infty, 1)$: EL
  - $\rho(v) = -exp(v)$, $\mathcal{V} = \Re$: ET

Brief Overview of Time Series GEL Estimation cont'd

GEL Estimator, $\hat{\theta}_{GEL}$: Solution to saddle point problem

Inner Loop:

- $\hat{\lambda}(\theta) = \arg \sup_{\lambda \in \Lambda_T(\theta)} \hat{P}_\rho(\theta, \lambda)$

  $= \arg \sup_{\lambda \in \Lambda_T(\theta)} \sum_{t=1}^{T} [\rho(\kappa \lambda' g_{tT}(\theta)) - \rho(0)]/T$

  FOC: $T^{-1} \sum_{t=1}^{T} \rho_1(k \hat{\lambda}(\theta)' g_{tT}(\theta)) g_{tT}(\theta) = 0$

Outer Loop:

- $\hat{\theta}_{GEL} = \arg \min_{\theta \in \Theta} \hat{P}_\rho(\theta, \hat{\lambda}(\theta)) = \arg \min_{\theta \in \Theta} \sup_{\lambda \in \Lambda_T(\theta)} \hat{P}_\rho(\theta, \lambda)$

  FOC: $T^{-1} \sum_{t=1}^{T} \rho_1(k \hat{\lambda}(\hat{\theta})' g_{tT}(\hat{\theta})) \partial g_{tT}(\hat{\theta}) / \partial \theta' \hat{\lambda}(\hat{\theta}) = 0$

Brief Overview of Time Series GEL Estimation cont'd

- If conditions on $\rho(v)$ are satisfied, and
- $\lambda(\theta) = \arg \sup_{\lambda \in \hat{\Lambda}_T(\theta)} \hat{P}_\rho(\theta, \lambda)$ exists, then for $t = 1, \cdots, T$,
  - $\pi_t(\hat{\theta}, \hat{\lambda}) = \frac{\rho_1(\kappa \hat{\lambda}' g_{tT}(\hat{\theta}))}{\sum_{t=1}^{T} \rho_1(\kappa \hat{\lambda}' g_{tT}(\hat{\theta}))}$: GEL implied probabilities
  - when pop mom condns hold, mom condns hold in sample
  - Potential Problem: Obtaining non-negative $\pi_t(\hat{\theta}, \hat{\lambda})$ requires $\kappa \hat{\lambda}' g_{tT}(\hat{\theta})$ be small uniformly in t; may not hold
  - Effective Solution: Use shrinkage estimator of Antoine, Bonnal & Renault (2007, Journal of Econometrics):

    $$\pi_t^*(\hat{\theta}, \hat{\lambda}) = \frac{1}{1+\varepsilon_T(\hat{\theta}, \hat{\lambda})} \pi_t(\hat{\theta}, \hat{\lambda}) + \frac{\varepsilon_T(\hat{\theta}, \hat{\lambda})}{1+\varepsilon_T(\hat{\theta}, \hat{\lambda})} \frac{1}{T}$$

    where $\varepsilon_T(\hat{\theta}, \hat{\lambda}) = -T \min\left[\min_{1 \leq t \leq T} \pi_t(\hat{\theta}, \hat{\lambda}), 0\right]$

Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ Tests of Over-ID Mom Conds

- Let $E[g(z_t, \theta_0)] = 0$ (θ_0 unique) $\Rightarrow$ Strong ID of θ_0
  - $g : q \times 1$
  - $\theta_0 : p \times 1$
  - $q > p \Rightarrow (q - p)$ Over-ID mom condns $\rightarrow$ **Test of Over-ID**
- Duality: $E[g(z_t, \theta_0)] = 0 \Leftrightarrow \lambda = 0$
- Exploiting above duality, Smith (2011, Econometric Theory) tests $H_0 : \lambda = 0$ against $H_a : \lambda \neq 0$ by developing
  - $\mathcal{LM} = (T/S_T^2)\hat{\lambda}'\hat{\Omega}_T(\hat{\theta})\hat{\lambda} \xrightarrow{d} \chi^2(q-p)$
  - $\mathcal{LR} = 2(T/S_T)\hat{P}_\rho(\hat{\theta}, \hat{\lambda})/(k_1^2/k_2) \xrightarrow{d} \chi^2(q-p)$
  - $\mathcal{S} = T\hat{g}_T(\hat{\theta})'\hat{\Omega}_T(\hat{\theta})^{-1}\hat{g}_T(\hat{\theta})/(k_1^2) \xrightarrow{d} \chi^2(q-p)$

My $C(\alpha)$-type LM ($\mathcal{LM}_\lambda$) Test of Over-ID Mom Conds

Let

- $\hat{\theta}$: $\sqrt{T}$-consistent est of θ_0, e.g., eff 2SGMM or CUGMM
- $\underset{q\times 1}{D_\lambda(\hat{\theta})}$, $\underset{p\times 1}{D_\theta(\hat{\theta})}$: score (gradient) w.r.t. λ and θ, respectively
- $\underset{(q+p)\times(q+p)}{D(\hat{\theta})} = \begin{pmatrix} \underset{q\times q}{D_{\lambda\lambda}(\hat{\theta})} & \underset{q\times p}{D_{\lambda\theta}(\hat{\theta})} \\ \underset{p\times q}{D_{\theta\lambda}(\hat{\theta})} & \underset{p\times p}{D_{\theta\theta}(\hat{\theta})} \end{pmatrix}$: Hessian w.r.t. λ and θ, or matrix of outer product of scores

Then, for testing $H_0 : \lambda = 0$ against $H_a : \lambda \neq 0$, my proposed $C(\alpha)$-type LM statistic:

- $$\begin{aligned} \mathcal{LM}_\lambda(\hat{\theta}) = \tfrac{T}{S_T}\tfrac{\kappa_2}{\kappa_1^2}\big(D_\lambda(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta})D_{\theta\theta}(\hat{\theta})^{-1}D_\theta(\hat{\theta})\big)' \\ \times\big(D_{\lambda\lambda}(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta})D_{\theta\theta}(\hat{\theta})^{-1}D_{\theta\lambda}(\hat{\theta})\big)^{-1} \\ \times\big(D_\lambda(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta})D_{\theta\theta}(\hat{\theta})^{-1}D_\theta(\hat{\theta})\big) \end{aligned}$$

Assumptions for Derivation of Limiting Distribution of $\mathcal{LM}_\lambda(\hat{\theta})$ Statistic

Assumption 1. The process $\{z_t\}_{t=1}^{\infty}$ is a finite dimensional stationary and strong mixing with mixing coefficients $\sum_{i=1}^{\infty} i^2 \alpha(i)^{(\nu-1)/\nu} < \infty$ for some $\nu > 1$.

Assumption 2. **(a)** $S_T \to \infty$ and $S_T = O(T^{\frac{1}{2}-\eta})$ for $\frac{1}{6} < \eta < \frac{1}{2}$; **(b)** $k(.) : \mathcal{R} \to [-k_{max}, k_{max}], k_{max} < \infty, k(0) \neq 0, k_1 \neq 0$, and is continuous at 0 and almost everywhere; **(c)** $\int_{(-\infty,\infty)} \overline{k}(x)dx < \infty$; **(d)** $|K(\lambda)| \geq 0$

$\forall \lambda \in \mathcal{R}$, where $\overline{k}(x) = \begin{cases} sup_{y \geq x}|k(y)| \text{ if } x \geq 0 \\ sup_{y \leq x}|k(y)| \text{ if } x < 0 \end{cases}$

and $K(\lambda) = \frac{1}{2\pi} \int k(x) e^{-i\lambda x} dx$.

Assumptions for Derivation of Limiting Distribution of $\mathcal{LM}_\lambda(\hat{\theta})$ Statistic cont'd

Assumption 3. **(a)** $\theta_0 \in \Theta$ is unique solution to $E[g_t(\theta)] = 0$; **(b)** Θ is compact; **(c)** $g_t(\theta)$ is continuous at each $\theta \in \Theta$ with probability one;
(d) $E[sup_{\theta\in\Theta}\|g_t(\theta)\|^\alpha] < \infty$, $\gamma > max(4\nu, \frac{1}{\eta})$;
(e) $\Omega(\theta) = \lim_{T\to\infty} var[T^{1/2}\hat{g}(\theta)]$ finite and p.d. $\forall\theta \in \Theta$.

Assumption 4. **(a)** $\rho(v) : \mathcal{V} \to \Re$ scalar, concave, $C^2 \in \mathcal{N}_0$, $\mathcal{V}$ open int in $\Re$ containing zero, $\partial\rho(0)/\partial v = \partial^2\rho(0)/\partial v^2 = -1$;
(b) $\lambda \in \Lambda_T = \{\lambda : \|\lambda\| \leq D(T/S_T^2)^{-\zeta}\}$, $D > 0$ and $\frac{1}{2\gamma\eta} < \zeta < \frac{1}{2}$.

Assumption 5. **(a)** $\theta_0 \in int(\Theta)$; **(b)** $g(., \theta)$ differentiable in $\mathcal{N}_{\theta_0}$ and $E[sup_{\theta\in\mathcal{N}_{\theta_0}}\|\partial g_t(\theta)/\partial\theta'\|^{\gamma/(\gamma-1)}] < \infty$; **(c)** $rank(G) = p$
where $G = E[\partial g_t(\theta_0)/\partial\theta']$.

Limiting Distribution of $\mathcal{LM}_\lambda(\hat{\theta})$ Statistic

Theorem 1 Let Assumptions 1-5 hold and let $\hat{\theta}$ be an efficient 2SGMM (therefore, $\sqrt{T}$-consistent) estimator of θ_0, based on kernel-smoothed moment indicator vectors. Then, under $H_0 : \lambda = 0$,

$$\mathcal{LM}_\lambda(\hat{\theta}) \xrightarrow{d} \chi^2_{q-p}.$$

Reject H_0 in favor of H_a at level α if $\mathcal{LM}_\lambda(\hat{\theta}) > \chi^2_{q-p,\,1-\alpha}$, where $\chi^2_{q-p,\,1-\alpha}$ is $(1-\alpha)$-th quantile of χ^2 dist with $q-p$ d.f.

Monte Carlo Study of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests

Objective of Monte Carlo Study:

- Investigate: size property of tests using
  - EL as representative of GEL class
  - 10,000 Monte Carlo replications
  - truncated $\kappa(\cdot)$
  - implied probabilities

Monte Carlo Design:

- Simple Linear IV Model based on
  - stationary TS data
  - w/ structural form errors and instruments as AR(1) processes
  - w/o exogenous variables in structural equation

Monte Carlo Design of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests cont'd

DGP:

- $y_t = \alpha x_{1t} + \beta x_{2t} + u_t, \quad u_t = \rho_u u_{t-1} + \varepsilon_{ut}$
- $x_t = (x_{1t}, x_{2t})' = \pi z_t + \varepsilon_{xt}, \quad z_t = \rho_z I_q z_{t-1} + \varepsilon_{zt}$

$$(t = 1, \cdots, T)$$

- z_t drawn independently of u_t and ε_{xt}
- $\alpha_0 = 0$ and $\beta_0 = 1$
- $q = 4$
- $\pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Monte Carlo Design of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests cont'd

- $\rho_u = \rho_z = 0.9$
- $\begin{pmatrix} \varepsilon_{ut} \\ \varepsilon_{xt} \end{pmatrix} = \begin{pmatrix} \varepsilon_{ut} \\ \varepsilon_{x_1 t} \\ \varepsilon_{x_2 t} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.3 \\ 0.8 & 0.3 & 1 \end{pmatrix}\right)$
- $\varepsilon_{zt} \sim N(0, I_4)$
- $T \in \{50, 100, 200, 500\}$
- $S_T \in \{1, 2, \cdots, 35\}$

Monte Carlo Results of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests

Results:

- Report P value plots in Figs 1-20
- Horizontal axis: probability values; Vertical axis: actual sizes of test statistic
- If P value plot lies close to 45° line, then it indicates hypothesis under test rejected approx correct proportion of time

  $\Rightarrow$ finite sample behavior of test stat well approx by its limiting dist
- Figs 1-12: P value plots of Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ stats

Monte Carlo Results of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests cont'd

Results cont'd:

- Their inspection reveals: For all sample sizes and bandwidths considered, all 3 tests, i.e., $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$, are oversized
  - However, they exhibit the desirable pattern that increase in sample size along with concomitant increase in bandwidth makes them less and less oversized
  - E.g., plots for T = 200 and ST $\in \{6, \cdots, 10\}$ are closer to 45° line than those for T = 100 and ST $\in \{6, \cdots, 10\}$, and plots for T = 500 and ST $\in \{11, \cdots, 15\}$ are even closer to 45° line than those for T = 200 and ST $\in \{6, \cdots, 10\}$

    $\Rightarrow$ for larger sample size and concomitant larger bandwidth, quality of asy approx to finite sample behavior of $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ stats becomes more satisfactory

Monte Carlo Results of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests cont'd/ Conclusion

Results cont'd:

- Figs 13-16 and Figs 17-20: P value plots of my $\mathcal{LM}_\lambda$ stat for testing Over-ID mom condns under same parameter configurations as for Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ stats
- Difference between settings in Figs 13-16 and Figs 17-20: In Figs 13-16 eff info obtained from Hessian, while that in Figs 17-20 obtained from OP of scores
- Inspection of Figs 13-16 and Figs 17-20 reveals: For all sample sizes and bandwidths considered, P value plots of my $\mathcal{LM}_\lambda$ stat lie closer to 45° line than those of Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ stats. Particularly at smaller prob values (thus more relevant for hypothesis testing)
  $\Rightarrow$ **Conclusion: $\mathcal{LM}_\lambda$ test enjoys superior/competitive size property relative to that of Smith's $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ tests**

Further Work To Do

Further Monte Carlo study to investigate:

- Power property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ tests
- Sensitivity of size and power properties of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ tests to other choices of
  - kernel functions
  - autoregressive parameters, and
  - length of instrument vectors

Thank You!