C(α)-type LM Test of Over-Identifying Moment Conditions in Time Series GEL Model

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Outline

1. Motivation for GEL Estimation & C(α)-type LM Test of Over-ID
2. Contributions of C(α)-type LM (LM$_{\lambda}$) Test Developed in this Paper
3. Brief Mention of My 3 C(α)-type LM (LM$_{\alpha}$, LM$_{\alpha}^w$, LM$_{\alpha\beta}^w$) Tests of Sub-Vector of Parameters under Strong & Weak ID in Time Series GEL Model Developed in 3 other Papers
4. Brief Overview of Time Series GEL Estimation
6. My LM$_{\lambda}$ Test of Over-ID Moment Conditions
7. Monte Carlo Study of Size Property of LM, LR, S, and LM$_{\lambda}$ Tests
8. Conclusion/Further Work To Do
Motivation for GEL Estimation: Consumption Based Capital Asset Pricing Model (CCAPM)

- Intertemporal optimization
  \[ \text{Max } E[\sum_{i=0}^{\infty} \delta_0^i U(c_{t+i})|\mathcal{I}_t] \text{ subject to} \]
  \[ c_t + \sum_{j=1}^{N} p_{j,t} q_{j,t} = w_t + \sum_{j=1}^{N} r_{j,t} q_{j,t} - m_j: \text{ budget constraint} \]
  \[ 0 \leq \delta_0 \leq 1: \text{ discount factor; } \mathcal{I}_t: \text{ info set} \]

- F.O.C. is given by the Euler equation
  \[ p_{j,t} U'(c_t) = E[\delta_0^{m_j} r_{j,t+m_j} U'(c_{t+m_j})|\mathcal{I}_t] \]

  \[ \Rightarrow \text{ value of utility lost by foregoing consumption in period } t \text{ to purchase 1 unit of asset } j = E(\text{discounted value of utility gained from consuming the return on investment in period } t + m_j) \]
Motivation cont’d

- **CRRA**: \( U(c_t) = \frac{c_t^{1-\gamma_0}}{1-\gamma_0} \) (\( \gamma_0 \): risk aversion parameter)

  \[ E[\delta_0^{m_j}(\frac{r_{j,t+m_j}}{p_{j,t}})(\frac{c_t+m_j}{c_t})^{\gamma_0}|I_t] - 1 = 0 \]

  - 2 parameters to estimate: \( \theta_0 \equiv (\delta_0, \gamma_0) \)

  - Let \( g(\cdot, \theta_0) = \delta_0^{m_j}(\frac{r_{j,t+m_j}}{p_{j,t}})(\frac{c_t+m_j}{c_t})^{\gamma_0} - 1 \)

  - By iterated conditional expectation

    \[ E[g(\cdot, \theta_0)z_t] = E[[Eg(\cdot, \theta_0)[I_t]z_t]] = 0 \] for any \( z_t \in I_t \)

  - E.g., \( z_t = \frac{r_{j,t}}{p_{j,t-m_j}}, \frac{c_t}{c_t-m_j} \) for \( j = 1, 2, \cdots, N; \)

    and any macro/finance variable \( \in I_t \)
Motivation cont’d

- \( j = 1 \) & \( m_j = 1 \) \( \Rightarrow \) \( g(\cdot, \theta_0) = \delta_0\left(\frac{r_{t+1}}{p_t}\right)^{\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma_0}} - 1 \)
  \( \Rightarrow \theta_0 \equiv (\delta_0, \gamma_0) \Rightarrow 2 \) parameters

- \( z_t = (1, \frac{c_t}{c_{t-1}}, \frac{r_t}{p_{t-1}})' \Rightarrow 3 \) moment conditions

\[
E\left[\left(\delta_0\left(\frac{r_{t+1}}{p_t}\right)^{\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma_0}} - 1\right)(1)\right] = 0
\]
\[
E\left[\left(\delta_0\left(\frac{r_{t+1}}{p_t}\right)^{\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma_0}} - 1\right)\left(\frac{c_t}{c_{t-1}}\right)\right] = 0
\]
\[
E\left[\left(\delta_0\left(\frac{r_{t+1}}{p_t}\right)^{\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma_0}} - 1\right)\left(\frac{r_t}{p_{t-1}}\right)\right] = 0
\]

\( \Rightarrow \) no. of moments, \( q = 3 \) \( > \) no. of parameters, \( p = 2 \)
\( \Rightarrow \) model over-identified
Estimation of Moment Condition Model: GMM & GEL

- \( q < p \) \( \Rightarrow \) \( \theta_0 \) under- or un-ID; not consistently estimable
- \( q \geq p \) \& \( \text{Rank} \left[ E \left( \frac{\partial g(\cdot, \theta_0)}{\partial \theta'} \right) \right] = p \) \( \Rightarrow \) \( \theta_0 \) consistently estimable
  - \( q = p \) \( \Rightarrow \) Just (Exact) ID \( \rightarrow \) MOM: sample moments = 0
    - i.e., \( g_T(\hat{\theta}) = \frac{1}{T} \sum_{t=1}^{T} g_t(\hat{\theta}) = 0 \)
- \( q > p \) \( \Rightarrow \) Over-ID \( \Rightarrow \) \( g_T(\hat{\theta}) \neq 0 \) \( \rightarrow \) GMM: minimize generalized distance of sample moments from zero
  - i.e., \( \hat{\theta} = \arg \min_{\theta} g_T(\theta)'W_T g_T(\theta) \)
  - \( W_T \): symmetric PD weighting matrix (distance metric)

- Limitation of GMM: Biased estimators in finite samples
- One of potential sources of bias: Estimated optimal \( W_T \)
Question: Could there be an alternative to GMM asymptotically equivalent to GMM, but does not require $W_T$?

Answer: Yes, GEL
Existing Tests of Over-ID & Objective of My Paper

- **Existing tests [Smith (2011, Econometric Theory)]:**
  - $\mathcal{LM}$, $\mathcal{LR}$, and $S$ tests of over-id moment conditions in TS GEL model
  - Have certain disadvantages

- **Objective of my paper:**
  - Develop $C(\alpha)$-type LM ($\mathcal{LM}_\lambda$) test of over-id moment conditions in TS GEL model
  - Thereby counter such disadvantages
Motivation for C(\(\alpha\))-type LM (\(\mathcal{LM}_\lambda\)) Test

- **Disadvantages of \(\mathcal{LM}, \mathcal{LR}, \text{ and } S\) tests:** They require estimation of GEL parameters, which
  
  - can be relatively difficult in nonlinear models
  - requires solution of a saddle-point problem
  - grows in complexity as \(p\) and/or \(q\) grow
  - therefore, computationally more involved
  - and hence, less appealing

- **C(\(\alpha\))-type LM (\(\mathcal{LM}_\lambda\)) test:** Attractive alternative when model is nonlinear
  
  - but \(\sqrt{T}\)-consistent estimators are relatively easily available
  - e.g., efficient 2SGMM or CUGMM, and one can, therefore,
  - take advantage of GEL implied probabilities w/o computing GEL estimators
  - therefore, computationally less involved
  - and hence, more appealing
Contributions of \( C(\alpha) \)-type LM (\( \mathcal{LM}_\lambda \)) Test Developed in this Paper

- Contributions of \( \mathcal{LM}_\lambda \) test
  - an attractive alternative and addition to the existing repertoire of Smith’s \( \mathcal{LM} \), \( \mathcal{LR} \), and \( S \) tests of over-id moment conditions in TS GEL model
  - possesses superior/competitive size property relative to that of Smith’s \( \mathcal{LM} \), \( \mathcal{LR} \), and \( S \) tests as evidenced by the Monte Carlo study completed thus far
Brief Mention of My 3 C(α)-type LM (LM_α, LM_w_α, and LM_w_αβ) Tests of Sub-Vector of Parameters under Strong & Weak ID in Time Series GEL Model Developed in 3 other Papers

The 3 tests of sub-vector of parameters under S & W ID developed in my 3 other papers are:

(i) \( LM_\alpha \) test: C(α)-type LM test of sub-vector \( \alpha \) when both the hypothesized sub-vector \( \alpha \) and the nuisance parameter vector \( \beta \) are S ID

(ii) \( LM_w_\alpha \) test: C(α)-type LM test of sub-vector \( \alpha \) when the hypothesized sub-vector \( \alpha \) contains both W & S ID parameters while the nuisance parameter vector \( \beta \) is S ID

(iii) \( LM_w_{\alpha\beta} \) test: C(α)-type LM test of sub-vector \( \alpha \) when both the hypothesized sub-vector \( \alpha \) and the nuisance parameter vector \( \beta \) contain W & S ID parameters
Brief Overview of Time Series GEL Estimation

Let

- \( \{z_t\}_{t=1}^{\infty} \): \( l \times 1 \) stationary strong mixing process
- \( \{z_t\}_{t=1}^{T} \): \( l \times 1 \) random sample
- \( \theta \): \( p \times 1 \) unknown parameters
- \( g(z_t, \theta) \equiv g_t(\theta) \): \( q \times 1 \) mom ind vectors, Borel funs
- \( q \geq p \)
- \( E[g(z_t, \theta_0)] = 0 \) (\( \theta_0 \) unique) \( \Rightarrow \) S ID of \( \theta_0 \)

Note: Dependence structure in TS data \( \rightarrow \)

- \( \text{Corr} \left[ g(z_t, \theta_0), \frac{\partial g(z_t, \theta_0)}{\partial \theta'} \right] \): one source of bias in estimators
- Question: How to remove this source of bias?
Answer: Use kernel-smoothed $g_t(\theta)$

$$g_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} \kappa\left(\frac{s}{S_T}\right) g_{t-s}(\theta)$$

- $\kappa(\cdot)$: kernel fun
- $S_T$: bandwidth

Anatolyev [(2005), Econometrica]: shows

- even when the moment indicator vectors are serially uncorrelated
- but not IID across time

smoothing the moment indicator vectors using kernel functions can reduce the bias
Brief Overview of Time Series GEL Estimation cont'd

To estimate $\theta$, construct GEL criterion fun:

$$\hat{P}_\rho(\theta, \lambda) = \sum_{t=1}^{T} \left[ \rho(\kappa \lambda' g_{tT}(\theta)) - \rho(0) \right] / T$$

- $\rho(v) : \mathcal{V} \to \mathbb{R}$; $\mathcal{V}$: open interval in $\mathbb{R}$ containing zero
  - $\rho(v)$: scalar fun, concave, $C^2$ in $\mathcal{N}_0$
  - $\partial \rho(0) / \partial v = \partial^2 \rho(0) / \partial v^2 = -1$
- $\kappa = \kappa_1 / \kappa_2$; $\kappa_j = \int_{-\infty}^{\infty} \kappa(a)^j da$, $j = 1, 2$
- $\Lambda_T(\theta) = \{ \lambda \in \mathbb{R}^q : \lambda' g_{tT}(\theta) \in \mathcal{V} \}$: Bounds on $\lambda$

- 3 common choices of $\rho(v)$:
  - $\rho(v) = -(1 + v)^2 / 2$, $\mathcal{V} = \mathbb{R}$: CU-GMM or EEL
  - $\rho(v) = \ln(1 - v)$, $\mathcal{V} = (-\infty, 1)$: EL
  - $\rho(v) = -\exp(v)$, $\mathcal{V} = \mathbb{R}$: ET
GEL Estimator, $\hat{\theta}_{GEL}$: Solution to saddle point problem

**Inner Loop:**

\[
\hat{\lambda}(\theta) = \arg \sup_{\lambda \in \Lambda_T(\theta)} \hat{P}_\rho(\theta, \lambda)
\]

\[
= \arg \sup_{\lambda \in \Lambda_T(\theta)} \sum_{t=1}^{T} [\rho(\kappa \lambda' g_t T(\theta)) - \rho(0)] / T
\]

**FOC:**

\[
T^{-1} \sum_{t=1}^{T} \rho_1(k \hat{\lambda}(\theta)' g_t T(\theta)) g_t T(\theta) = 0
\]

**Outer Loop:**

\[
\hat{\theta}_{GEL} = \arg \min_{\theta \in \Theta} \hat{P}_\rho(\theta, \hat{\lambda}(\theta)) = \arg \min_{\theta \in \Theta} \sup_{\lambda \in \Lambda_T(\theta)} \hat{P}_\rho(\theta, \lambda)
\]

**FOC:**

\[
T^{-1} \sum_{t=1}^{T} \rho_1(k \hat{\lambda}(\hat{\theta})' g_t T(\hat{\theta})) \partial g_t T(\hat{\theta}) / \partial \theta' \hat{\lambda}(\theta) = 0
\]
If conditions on $\rho(v)$ are satisfied, and

$$\lambda(\theta) = \arg \sup_{\lambda \in \hat{\Lambda}_T(\theta)} \hat{P}_\rho(\theta, \lambda)$$

exists, then for $t = 1, \cdots, T$,

$$\pi_t(\hat{\theta}, \hat{\lambda}) = \frac{\rho_1(\kappa' g_{tT}(\hat{\theta}))}{\sum_{t=1}^{T} \rho_1(\kappa' g_{tT}(\hat{\theta}))}$$

: GEL implied probabilities

when pop mom condns hold, mom condns hold in sample

Potential Problem: Obtaining non-negative $\pi_t(\hat{\theta}, \hat{\lambda})$ requires $\kappa' g_{tT}(\hat{\theta})$ be small uniformly in $t$; may not hold


$$\pi_t^*(\hat{\theta}, \hat{\lambda}) = \frac{1}{1 + \varepsilon_T(\hat{\theta}, \hat{\lambda})} \pi_t(\hat{\theta}, \hat{\lambda}) + \frac{\varepsilon_T(\hat{\theta}, \hat{\lambda})}{1 + \varepsilon_T(\hat{\theta}, \hat{\lambda})} \frac{1}{T}$$

where $\varepsilon_T(\hat{\theta}, \hat{\lambda}) = -T \min \left[ \min_{1 \leq t \leq T} \pi_t(\hat{\theta}, \hat{\lambda}), 0 \right]$
Smith’s $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ Tests of Over-ID Mom Conds

- Let $E[g(z_t, \theta_0)] = 0$ ($\theta_0$ unique) $\Rightarrow$ Strong ID of $\theta_0$
  - $g : q \times 1$
  - $\theta_0 : p \times 1$
  - $q > p \Rightarrow (q - p)$ Over-ID mom condns $\rightarrow$ Test of Over-ID

- Duality: $E[g(z_t, \theta_0)] = 0 \iff \lambda = 0$

- Exploiting above duality, Smith (2011, Econometric Theory) tests $H_0 : \lambda = 0$ against $H_a : \lambda \neq 0$ by developing
  - $\mathcal{LM} = (T/S_T^2)\hat{\lambda}'\hat{\Omega}_T(\hat{\theta})\hat{\lambda} \xrightarrow{d} \chi^2(q - p)$
  - $\mathcal{LR} = 2(T/S_T)\hat{P}_\rho(\hat{\theta}, \hat{\lambda})/(k_1^2/k_2) \xrightarrow{d} \chi^2(q - p)$
  - $\mathcal{S} = T\hat{g}_T(\hat{\theta})'\hat{\Omega}_T(\hat{\theta})^{-1}\hat{g}_T(\hat{\theta})/(k_1^2) \xrightarrow{d} \chi^2(q - p)$
My $C(\alpha)$-type LM ($\mathcal{LM}_\lambda$) Test of Over-ID Mom Conds

Let

- $\hat{\theta} : \sqrt{T}$-consistent est of $\theta_0$, e.g., eff 2SGMM or CUGMM
- $D_{\lambda}(\hat{\theta})$, $D_{\theta}(\hat{\theta})$: score (gradient) w.r.t. $\lambda$ and $\theta$, respectively
- $D(\hat{\theta}) = \begin{pmatrix} D_{\lambda\lambda}(\hat{\theta}) & D_{\lambda\theta}(\hat{\theta}) \\ D_{\theta\lambda}(\hat{\theta}) & D_{\theta\theta}(\hat{\theta}) \end{pmatrix}$: Hessian w.r.t. $\lambda$ and $\theta$, or matrix of outer product of scores

Then, for testing $H_0 : \lambda = 0$ against $H_a : \lambda \neq 0$, my proposed $C(\alpha)$-type LM statistic:

- $\mathcal{LM}_\lambda(\hat{\theta}) = \frac{T}{S_T \kappa_1^2 \kappa_2^2} \left( D_{\lambda}(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta}) D_{\theta\theta}(\hat{\theta})^{-1} D_{\theta}(\hat{\theta}) \right)' \times \left( D_{\lambda}(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta}) D_{\theta\theta}(\hat{\theta})^{-1} D_{\theta}(\hat{\theta}) \right)^{-1} \times \left( D_{\lambda}(\hat{\theta}) - D_{\lambda\theta}(\hat{\theta}) D_{\theta\theta}(\hat{\theta})^{-1} D_{\theta}(\hat{\theta}) \right)$
Assumptions for Derivation of Limiting Distribution of $\mathcal{LM}_\lambda(\hat{\theta})$ Statistic

**Assumption 1.** The process $\{z_t\}_{t=1}^\infty$ is a finite dimensional stationary and strong mixing with mixing coefficients $\sum_{i=1}^\infty i^2 \alpha(i)^{(\nu-1)/\nu} < \infty$ for some $\nu > 1$.

**Assumption 2.** (a) $S_T \to \infty$ and $S_T = O(T^{1/2-\eta})$ for $1/6 < \eta < 1/2$; (b) $k(.) : \mathcal{R} \to [-k_{max}, k_{max}], k_{max} < \infty, k(0) \neq 0, k_1 \neq 0$, and is continuous at 0 and almost everywhere; (c) $\int_{(-\infty,\infty)} \overline{k}(x) dx < \infty$; (d) $|K(\lambda)| \geq 0$ for all $\lambda \in \mathcal{R}$, where $\overline{k}(x) = \left\{ \begin{array}{ll} \sup_{y \geq x} |k(y)| & \text{if } x \geq 0 \\ \sup_{y \leq x} |k(y)| & \text{if } x < 0 \end{array} \right.$

and $K(\lambda) = \frac{1}{2\pi} \int k(x)e^{-i\lambda x} dx$. 
Assumptions for Derivation of Limiting Distribution of $\mathcal{LM}_\lambda(\hat{\theta})$ Statistic cont’d

**Assumption 3.** (a) $\theta_0 \in \Theta$ is unique solution to $E[g_t(\theta)] = 0$; (b) $\Theta$ is compact; (c) $g_t(\theta)$ is continuous at each $\theta \in \Theta$ with probability one; (d) $E[\sup_{\theta \in \Theta} \|g_t(\theta)\|^\alpha] < \infty$, $\gamma > \max(4\nu, \frac{1}{\eta})$; (e) $\Omega(\theta) = \lim_{T \to \infty} \text{var}[T^{1/2} \hat{g}(\theta)]$ finite and p.d. $\forall \theta \in \Theta$.

**Assumption 4.** (a) $\rho(v) : \mathcal{V} \to \mathbb{R}$ scalar, concave, $C^2 \in \mathcal{N}_0$, $\mathcal{V}$ open int in $\mathbb{R}$ containing zero, $\partial \rho(0)/\partial v = \partial^2 \rho(0)/\partial v^2 = -1$; (b) $\lambda \in \Lambda_T = \{\lambda : \|\lambda\| \leq D(T/S_T^2)^{-\zeta}\}$, $D > 0$ and $\frac{1}{2\gamma\eta} < \zeta < \frac{1}{2}$.

**Assumption 5.** (a) $\theta_0 \in \text{int}(\Theta)$; (b) $g(., \theta)$ differentiable in $\mathcal{N}_{\theta_0}$ and $E[\sup_{\theta \in \mathcal{N}_{\theta_0}} \|\partial g_t(\theta)/\partial \theta'\|^{\gamma/(\gamma-1)}] < \infty$; (c) $\text{rank}(G) = p$ where $G = E[\partial g_t(\theta_0)/\partial \theta']$. 
Theorem 1 Let Assumptions 1-5 hold and let $\hat{\theta}$ be an efficient 2SGMM (therefore, \(\sqrt{T}\)-consistent) estimator of \(\theta_0\), based on kernel-smoothed moment indicator vectors. Then, under \(H_0: \lambda = 0\),

\[
\mathcal{LM}_\lambda(\hat{\theta}) \xrightarrow{d} \chi_{q-p}^2.
\]

Reject \(H_0\) in favor of \(H_a\) at level \(\alpha\) if \(\mathcal{LM}_\lambda(\hat{\theta}) > \chi_{q-p, 1-\alpha}^2\), where \(\chi_{q-p, 1-\alpha}^2\) is \((1 - \alpha)\)-th quantile of \(\chi^2\) dist with \(q - p\) d.f.
Monte Carlo Study of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $S$, and $\mathcal{LM}_\lambda$ Tests

Objective of Monte Carlo Study:

- Investigate: size property of tests using
  - EL as representative of GEL class
  - 10,000 Monte Carlo replications
  - truncated $\kappa(\cdot)$
  - implied probabilities

Monte Carlo Design:

- Simple Linear IV Model based on
  - stationary TS data
  - w/ structural form errors and instruments as AR(1) processes
  - w/o exogenous variables in structural equation
Monte Carlo Design of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests cont’d

DGP:

- $y_t = \alpha x_{1t} + \beta x_{2t} + u_t, \quad u_t = \rho_u u_{t-1} + \varepsilon_{ut}$
- $x_t = (x_{1t}, x_{2t})' = \pi z_t + \varepsilon_{xt}, \quad z_t = \rho_z I_q z_{t-1} + \varepsilon_{zt}$

$(t = 1, \cdots, T)$

- $z_t$ drawn independently of $u_t$ and $\varepsilon_{xt}$
- $\alpha_0 = 0$ and $\beta_0 = 1$
- $q = 4$
- $\pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
Monte Carlo Design of $\mathcal{LM}$, $\mathcal{LR}$, $S$, and $\mathcal{LM}_\lambda$ Tests cont’d

- $\rho_u = \rho_z = 0.9$

- \[
\begin{pmatrix}
\varepsilon_{ut} \\
\varepsilon_{xt}
\end{pmatrix}
= \begin{pmatrix}
\varepsilon_{ut} \\
\varepsilon_{x1t} \\
\varepsilon_{x2t}
\end{pmatrix} \sim N \left( 0, \begin{pmatrix}
1 & 0.8 & 0.8 \\
0.8 & 1 & 0.3 \\
0.8 & 0.3 & 1
\end{pmatrix} \right)
\]

- $\varepsilon_{zt} \sim N(0, I_4)$

- $T \in \{50, 100, 200, 500\}$

- $S_T \in \{1, 2, \cdots, 35\}$

Monte Carlo Results of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $\mathcal{S}$, and $\mathcal{LM}_\lambda$ Tests

Results:

- Report P value plots in Figs 1-20
- Horizontal axis: probability values; Vertical axis: actual sizes of test statistic
- If P value plot lies close to 45° line, then it indicates hypothesis under test rejected approx correct proportion of time
  $\Rightarrow$ finite sample behavior of test stat well approx by its limiting dist
- Figs 1-12: P value plots of Smith’s $\mathcal{LM}$, $\mathcal{LR}$, and $\mathcal{S}$ stats
Results cont’d:

- Their inspection reveals: For all sample sizes and bandwidths considered, all 3 tests, i.e., $\mathcal{LM}$, $\mathcal{LR}$, and $S$, are oversized.
  - However, they exhibit the desirable pattern that increase in sample size along with concomitant increase in bandwidth makes them less and less oversized.
  - E.g., plots for $T = 200$ and $ST \in \{6, \cdots, 10\}$ are closer to $45^\circ$ line than those for $T = 100$ and $ST \in \{6, \cdots, 10\}$, and plots for $T = 500$ and $ST \in \{11, \cdots, 15\}$ are even closer to $45^\circ$ line than those for $T = 200$ and $ST \in \{6, \cdots, 10\}$.

$\Rightarrow$ for larger sample size and concomitant larger bandwidth, quality of asy approx to finite sample behavior of $\mathcal{LM}$, $\mathcal{LR}$, and $S$ stats becomes more satisfactory.
Monte Carlo Results of Size Property of $\mathcal{LM}$, $\mathcal{LR}$, $S$, and $\mathcal{LM}_\lambda$ Tests cont’d/ Conclusion

Results cont’d:

- Figs 13-16 and Figs 17-20: P value plots of my $\mathcal{LM}_\lambda$ stat for testing Over-ID mom condns under same parameter configurations as for Smith’s $\mathcal{LM}$, $\mathcal{LR}$, and $S$ stats.

- Difference between settings in Figs 13-16 and Figs 17-20: In Figs 13-16 eff info obtained from Hessian, while that in Figs 17-20 obtained from OP of scores.

- Inspection of Figs 13-16 and Figs 17-20 reveals: For all sample sizes and bandwidths considered, P value plots of my $\mathcal{LM}_\lambda$ stat lie closer to 45° line than those of Smith’s $\mathcal{LM}$, $\mathcal{LR}$, and $S$ stats. Particularly at smaller prob values (thus more relevant for hypothesis testing).

$\Rightarrow$ Conclusion: $\mathcal{LM}_\lambda$ test enjoys superior/competitive size property relative to that of Smith’s $\mathcal{LM}$, $\mathcal{LR}$, and $S$ tests.
Further Work To Do

Further Monte Carlo study to investigate:

- **Power property** of $\mathcal{LM}$, $\mathcal{LR}$, $S$, and $\mathcal{LM}_\lambda$ tests

- **Sensitivity of size and power properties** of $\mathcal{LM}$, $\mathcal{LR}$, $S$, and $\mathcal{LM}_\lambda$ tests to other choices of
  - kernel functions
  - autoregressive parameters, and
  - length of instrument vectors
Thank You!