Quantitative Uniform Approximation by Generalized Discrete Singular Operators

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Abstract. Here we study the approximation properties with rates of generalized discrete versions of Picard, Gauss-Weierstrass, and Poisson-Cauchy singular operators. We treat both the unitary and non-unitary cases of the operators above. We establish quantitatively the pointwise and uniform convergences of these operators to the unit operator by involving the uniform higher modulus of smoothness of a uniformly continuous function.

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Key Words and Phrases: Discrete singular operator, modulus of smoothness, uniform convergence.

1 Introduction

This article is motivated mainly by "*Sur les multiplicateurs d'interpolation*, J. Math. Pures Appl., IX, 23(1944), 219-247", where J. Favard in 1944 introduced the discrete version of Gauss-Weierstrass operator

$$\left(F_n f\right)(x) = \frac{1}{\sqrt{\pi n}} \sum_{\nu=-\infty}^{\infty} f\left(\frac{\nu}{n}\right) \exp\left(-n\left(\frac{\nu}{n}-x\right)^2\right), \tag{1}$$

$n \in \mathbb{N}$, which has the property that $\left(F_n f\right)(x)$ converges to $f(x)$ pointwise for each $x \in \mathbb{R}$, and uniformly on any compact subinterval of $\mathbb{R}$, for each continuous function f ($f \in C(\mathbb{R})$) that fulfills $|f(t)| \leq Ae^{Bt^2}$, $t \in \mathbb{R}$, where A, B are positive constants.

The well-known Gauss-Weierstrass singular convolution integral operators is

$$(W_n f)(x) = \sqrt{\frac{n}{\pi}} \int_{-\infty}^{\infty} f(u) \exp\left(-n (u-x)^2\right) du. \tag{2}$$

We are also motivated by "G.A. Anastassiou, *Intelligent Mathematics: Computational Analysis*, Springer, Heidelberg, New York, USA, 2011", "G.A. Anastassiou, *Approximation by Discrete Singular Operators*, Cubo, Vol.15, No.1 (2013), 97-112.", and "G.A. Anastassiou and R.A. Mezei, *Approximation by Singular Integrals,*Cambridge Scientific Publishers, Cambrige, UK, 2012" where the authors studied extensively the approximation properties of particular generalized singular integral operators such as Picard, Gauss-Weierstrass, and Poisson-Cauchy as well as the general cases of singular integral operators.These operators are not necessarily positive linear operators.

In this article, we define the discrete versions of the operators mentioned above and we study quantitatively their uniform approximation properties regarding convergence to the unit. We examine thoroughly the unitary and non-unitary cases and their interconnections.

STABILITY ANALYSIS OF APPROXIMATE DYNAMIC PROGRAMMING (ADP) CONTROL

YURY SOKOLOV

The method of dynamic programming provides a recursive procedure for computing optimal solution to complex problems by breaking them into a sequence of simpler optimization subproblems. However, this poses significant challenges for high dimensional problems. In such cases, it is beneficial to approximate the objective (value) function that satisfies Bellman equation and get approximation of the optimal solution to the original problem.

In this talk, we provide stability results for ADP control by showing that our control approach is uniformly ultimately bounded under specific conditions for the parameters of the control design. Our approach provides improved stability results with respect to the state-of-art of ADP control.

On the Edge Spectrum of Saturation Number for Paths and Stars

Ali Dogan

Abstract

For a given graph H, we say that a graph G on n vertices is H-saturated if H is not a subgraph of G, but for any edge $e \in E(\bar{G})$ the graph $G+e$ contains a subgraph isomorphic to H. Let $Sat(n; H)$ be the set of all H-saturated graphs of order n. Let $sat(n; H)$ and $ex(n; H)$ denote the minimum and the maximum number of edges of a graph in $Sat(n; H)$, respectively. The set of all values m, where $sat(n; H) \leq m \leq ex(n; H)$, for which there exists an H-saturated graph on n vertices and m edges is called the edge spectrum for H-saturated graphs. In this talk we present some background and then discuss the edge spectrum of the saturation number for paths and stars.

HOW MANY TRIANGLES CAN A GRAPH HAVE?

KAMIL POPIELARZ

In this talk, we ask the following question: how many triangles can a graph on n vertices have? Surprisingly, very little is known about this problem. Let T_n be the set of possible number of triangles in a graph on n vertices. The first main result says that every natural number less than $\binom{n}{3} - \left(\sqrt{2} + o(1)\right) n^{3/2}$ belongs to T_n. On the other hand, we show that there is a number $m = \binom{n}{3} - \left(\sqrt{2} + o(1)\right) n^{3/2}$ which is not a member of T_n. In addition, we prove that there are two interlacing sequences $\binom{n}{3} - \left(\sqrt{2} + o(1)\right) n^{2/3} = c_1 \leq d_1 \leq c_2 \leq d_2 \leq \cdots \leq c_s \leq d_s = \binom{n}{3}$ with $|c_t - d_t| = n - 2 - \binom{s-t+1}{2}$ such that $(c_t, d_t) \cap T_n = \emptyset$ for all t. Moreover, we obtain a generalization of the results for the set of possible number of copies of a connected graph H in a graph on n vertices.

Intuitionistic Models of Intuitionistic Logic

David Lewis

Abstract. In 1965, Kripke introduced possible-world semantics for intuitionistic logic and showed that intuitionistic predicate calculus was complete, however, the proof was not constructive. In 1973, Veldman proved that intuitionistic predicate calculus was complete with respect to modified Kripke models, which admit impossible or strange worlds where every sentence is true. Modified Kripke models are classically equivalent to ordinary ones because one can simply ignore the impossible worlds, however, in an intuitionist setting, it is not necessarily the case that all worlds are either possible or impossible. In an ordinary Kripke model, validity for negations is defined as follows: ~P is valid at a world W if there are no possible worlds accessible from W in which P is valid. In a modified Kripke model, it can instead be defined as: ~P is valid at a world W if all worlds accessible in which P is valid are impossible. These conditions are classically equivalent, but, intuitionistically, the second one is stronger.

On a Problem of Littlewood and Extremely Sperse Partition-regular Patterns

Julian Sahasrabudhe

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In his 1968 monograph entitled "Some Problems in Real and Complex Analysis", J.E. Littlewood considered the minimum number of real roots that a *cosine polynomial* of the form

$$f_A(\theta) = \sum_{a \in A} \cos(a\theta)$$

could have in the interval $[0, 2\pi]$, where A is a finite set of integers. He conjectured that the number of roots of such polynomials would be "$|A| - 1$, or not much less". This conjecture was later disproved rather dramatically by P. Borwein, T. Erdelyi, R. Ferguson, and R. Lockhart, who showed the existence of cosine polynomials with as few as $C|A|^{5/6}\log(|A|)$ roots. Later, Peter Borwein and Tamás Erdély, showed that the number of roots of such f_A tend to infinity as $|A| \to \infty$, under some additional (rather strong) assumptions. I finally give an unconditional lower bound for the number of zeros of a cosine polynomial. In particular, I prove that if $R \subseteq \mathbb{R} \setminus \{0\}$ is a finite set then the number of zeros of a cosine polynomial

$$f_A(\theta) = \sum_{a \in A} C_a \cos(a\theta),$$

where $C_a \in R$, tend to infinity with $|A|$. Indeed, this theorem also address the analogous question for $\{-1, +1\}$ polynomials which has received independent attention. It was shown by K. Mukunda that that such polynomials have at least 3 roots who was later topped by Paulius Drungilas, who showed they have 7.

It is perhaps worth noting that the proof introduces some new ideas to the study of cosine polynomials. It uses (basic) algebraic tools along side Fourier analytic methods.

Ramsey theory on the integers looks to understand what arithmetic structure is preserved after the integers are partitioned arbitrarily into a finite number of pieces. Indeed, at first blush, it is not clear that *anything* can be said about the arithmetic structure of one of the parts of an arbitrary partition. The first result in this direction was proved by Schur, who showed that an arbitrary partition of the integers admits a triple a, b, c such that a, b, c all lie within a class and $a + b = c$. While the theory of linear equations is now well understood, the theory of non-linear equations appears to be much more difficult. Contributing to this theory, the author has proved that every partition of the integers into finitely many classes contains $a, b > 1$ such that a, b, a^b all lie within a class. The author goes on to show a much greater class of patterns defined by exponentials can also be found within a class of a given finite partition. This work answers a question of A. Sisto who showed that every 2-colouring of $\mathbb{N}$ contains a monochromatic triple a, b, a^b and the work of T. Brown who further explored what exponential patterns are present in every 2-colouring.

An n vertex graph G is said to be k-universal if for every collection of vertices $x_1, \ldots, x_k$ and $y_1, \ldots, y_k$ (all distinct from the x_i) we can find a vertex v such that v that is adjacent to all of $x_1, \ldots, x_k$ and none of $y_1, \ldots, y_k$. Universal graphs are naturally arising objects in combinatorics as they contain an induced copy of every fixed graph H on at most k vertices. While it is easy to establish that k universal graphs exist for $k \leq c\log(n)$, the existence of *triangle free* universal graphs remains somewhat of a mystery. An n-vertex graph G is said to be a triangle free-universal graph if it is triangle free and if for every $x_1, \ldots, x_k$ and $y_1, \ldots, y_k$ where the y_i are distinct from the x_i *and* if there are no edges among $x_1, \ldots, x_k$ there exists a vertex v that are joined to all of $x_1, \ldots, x_k$ and none of the y_i. Amazingly, it is not known if triangle free graphs exist for arbitrary large k. Letzter and the author show that there do no exist any triangle free universal graphs with $k \geq C\frac{\log(n)}{\log\log n}$, thus providing the first non-trivial progress on this problem and first concrete evidence that the triangle free version of the problem is fundamentally different form the version of the problem without the triangle-free restriction.

The author also has forthcoming work in graph theory with Bhargav Narayanan and Istvan Tomon; in Bootstrap Percolation with Béla Bollobá, Michał Przykucki and Oliver Riordan; and independent papers on Combinatorial optimization and graph colouring.