Masters Exam (Spring 2003)
Algebra I

Answer any **three** of the following six questions.
You should state clearly any general results you use.

1. Classify all groups of order $45 = 3^2.5$ stating clearly any results that you use.
2. Let A be an abelian group and let $D = \{(a, a) : a \in A\}$ be the set of 'diagonal elements' in $A \times A$. Show that D is a subgroup of $A \times A$ and that $(A \times A)/D \cong A$.
3. Let $f\colon G \to G$ be given by $f(x) = x^2$. Show that f is a homomorphism if and only if G is abelian.
4. Let R be a commutative ring with 1.
   (a) Show that every maximal ideal is prime.
   (b) Show that if R is a PID then every non-zero prime ideal is maximal.
5. A *Boolean ring* is a ring in which $x^2 = x$ for all x. Let R be a commutative Boolean ring (with 1).
   (a) Show that $2x = 0$ for all $x \in R$.
   (b) Show that every prime ideal of R is maximal.
6. Find the gcd of the elements $5 + 7i$ and $3 - 5i$ in $\mathbb{Z}[i]$.