Master Exam-2006 Spring
Algebra II

Answer any three of the following five problems.

1. Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 - 2$.

2. Let F be a finite field. Show that there is a prime p such that $p \cdot x = 0$ for any $x \in F$.

3. (Schur's Lemma) Recall an R-module M is called *irreducible* if $M \neq 0$ and if 0 and M are the only submodules of M. Show that if M is an irreducible R module, then any nonzero R-module homomorphism from M to M is an isomorphism.

4. Determine all possible Jordan canonical forms for a linear transformation with characteristic polynomial $(x-2)^3(x-3)^2$.

5. Let M be a (left) module over a ring R (not necessarily commutative), I a left ideal of R, and m a nonzero element in M. Show that $\{am : a \in I\}$ is a submodule of M.