MASTER EXAM-PART II (ALGEBRA) -SUMMER 2010

Do three of following problems.

1. A ring R is called a *Boolean ring* if $a^2 = a$ for all $a \in R$. Prove that every Boolean ring is commutative.

2. Show that every ideal in a Euclidean Domain is principal.

3. Let R be a commutative ring with identity and M an ideal of R. Show that M is a maximal ideal if and only if R/M is a field.

4. List all non-isomorphic ablian groups of order 2704.

5. (The second isomorphism theorem) Let N be a normal subgroup of a group G and A another subgroup of G. Show that AN is a subgroup of G and AN/N is isomorphic to $A/(A \cap N)$.

6. Let G be a finite group with order n, and H a subgroup of G such that n is not a divisor of $i_G(H)!$ where $i_G(H) = \frac{\circ(G)}{\circ(H)}$ and $\circ(H)$ is the order of the group H. Show that there is a normal subgroup N of G contained in H.