

Master's Paper: Math 7350

Spring 2003

Answer any two questions; Credit will be given only for the best two questions Show all working; State clearly all theorems that you apply

1. (a) Prove that $L^2(\mathbb{R})$ is not a subset of $L^1(\mathbb{R})$.
(b) Prove that $L^1(\mathbb{R})$ is not a subset of $L^2(\mathbb{R})$.

2. Let the sequence $(f_n)_{n=1}^{\infty}$ and the function f be real-valued functions on $[0, 1]$.
(a) Prove that if $\|f_n - f\|_1 \rightarrow 0$ then f_n converges to f in measure.
(b) Is it true that if $\|f_n - f\|_1 \rightarrow 0$ then $f_n(x) \rightarrow f(x)$ for almost every $x \in [0, 1]$.

3. (a) State the dominated convergence theorem.
(b) Starting from the monotone convergence theorem, give a proof of the dominated convergence theorem.