Master's Paper: Math 7350

Summer 2003

Answer any two questions; Credit will be given only for the best two questions Show all working; State clearly all theorems that you apply

1. For each part, prove the statement or give a counterexample.
   (a) If (X, μ) is a measure space, then for any nested sequence of sets $A_1 \supseteq A_2 \supseteq \ldots$, $\mu(\bigcap_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$.
   (b) If (X, μ) is a measure space, then for any nested sequence of sets $A_1 \subseteq A_2 \subseteq \ldots$, $\mu(\bigcup_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$.

2. Let (X, μ) be a measure space. Prove that if f is a strictly positive measurable function, then $\int f\, d\mu > 0$.

3. Suppose that f and $f_1, f_2, f_3, \ldots$ are measurable functions on $\mathbb{R}$.
   (a) State what it means to say that the sequence f_n converges to f in measure.
   (b) Prove that if f_n converges to f in measure, then there is a subsequence of the f_n that converges pointwise to f.