Master's Exam: Real Variables I

Department of Mathematical Sciences
The University of Memphis

January 21, 2006

Instructions: Solve any **two** of the following three problems. Show all work.

Problem 1

Argue carefully to show that, for $\alpha < 1$, the limit

$$\lim_{n\to\infty} \int_0^n \left(1 - \frac{x}{n}\right)^n e^{\alpha x} \sin x \, dx$$

exists. Hint: $1 - \frac{x}{n} \leq e^{-x/n}$ for all $x \geq 0$.

Problem 2 Define the Lebesgue outer measure μ^* on $\mathbb{R}$. Prove it is countable subadditive i.e. for any countable collection $\{A_i : i \in \mathbb{N}\}$ of subsets of $\mathbb{R}$,

$$\mu^*(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} \mu^*(A_i).$$

Problem 3 State the Monotone Convergence Theorem and use Fatou's Lemma to prove it.