

MASTER'S EXAM, REAL VARIABLES, JANUARY 2012

ANNA KAMIŃSKA

Choose two problems.

1. State and prove the Bolzano-Weierstrass Theorem.

2. Show that the function

$$f(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable on \mathbb{R} . Is f' continuous on \mathbb{R} ?

3. Let $f : [a, b] \rightarrow \mathbb{R}$ be a bounded function.

- (i) Define the Riemann integral of f over the interval $[a, b]$, that is $\int_a^b f(x) dx$.
- (ii) Prove using definition that

$$\int_a^b f(x) dx = \int_{-b}^{-a} f(-x) dx.$$