Master Exam-Part II (Real Analysis I)

1. State and prove the Fatou Lemma.
2. Let $f \geq 0$ be integrable on $\mathbb{R}$. Show that $F(x) = \int_{-\infty}^{x} f$ is continuous.
3. Let $f_n(x) = nx^{n-1} - (n+1)x^n$, $x \in (0,1)$. Show

$$\int_{(0,1)} \sum_{n=1}^{\infty} f_n \neq \sum_{n=1}^{\infty} \int_{(0,1)} f_n,$$

and

$$\sum_{n=1}^{\infty} \int_{(0,1)} |f_n| = \infty.$$