**Masters' exam: Complex Analysis**

Fall 2008

Solve one of the problems 1 and 2, and one of the problems 3 and 4.

**1.** (a) Suppose that an entire function is bounded by $M$ along $|z| = R$. Show that the coefficients $C_k$ in its power series expansion about 0 satisfy

$$|C_k| \leq \frac{M}{R^k}.$$

(b) Suppose that a polynomial is bounded by 1 in the unit disc. Sow that all its coefficients are bounded by 1.

**2.** (a) Classify the singularities of the function

$$f(z) = \frac{e^{\frac{1}{z^2}}}{z-1}.$$

(b) If $f$ is holomorphic on a deleted neighborhood of 0, and satisfies $|f(z)| > 1/|z|$ for all nonzero $z$ on the unit disc, what kind of singularity does $f$ have at 0?

**3.** Compute the integral

$$\int_0^\infty \frac{\cos x}{x^2+1}\, dx.$$

**4.** (a) Find a conformal mapping between the strip $\{z \mid 0 < \mathrm{Re}(z) < 1\}$ and the upper half-plane.

(b) Find a conformal mapping between the unit disc and the first quadrant.