

Masters Exam (Spring 2014)

Graph Theory

Answer any **four** of the following six questions.

You should state clearly any general results you use.

1. Recall that a *tournament* is an oriented complete graph. Show that any tournament has a hamiltonian directed path.
2. Let T be a tree with m edges. Show that any graph with minimum degree at least m contains T as a subgraph. Is this still true with minimum degree $m - 1$?
3. Prove that every connected graph of order $n > 1$ has at least two vertices which are not cut vertices.
4. Let G be a graph with n vertices and let \overline{G} be the edge complement of G . Prove *either* of the following two results:
 - (a) $\chi(G) + \chi(\overline{G}) \leq n + 1$,
 - (b) $\chi(G)\chi(\overline{G}) \geq n$.
5.
 - (a) State Hall's theorem.
 - (b) Deduce that any regular bipartite graph contains a perfect matching.
6. Suppose G is a connected graph of order n and $1 \leq k \leq n$. Show that G has a connected subgraph of order k .