Master's for Teachers Exam

Fall 2004

Answer question 1 from section A and any **two** *questions from section B.*
Time allowed: **2 hours**.

Section A

1. In Liping Ma's book, *Knowing and Teaching Elementary Mathematics*, scenarios in mathematics were posed to teachers in the United States and China. Two of the scenarios are listed below. Answer the questions that follow each of the scenarios.

   Scenario 1: $\mathbf{1\frac{3}{4} \div \frac{1}{2}}$

   (a) Find the following: $1\frac{3}{4} \div \frac{1}{2}$

   (b) Give two different ways of finding the answer to (a) other than inverting and multiplying.

   (c) Write a word problem for $1\frac{3}{4} \div \frac{1}{2}$. You need to devise a problem and solve the problem you wrote. A picture or diagram should be included in your explanation.

   (d) List 2 misconceptions students may have when solving the problem you wrote in part (c).

   Scenario 2: A seventh grade student announces she has discovered a new (to her) mathematical theorem. "If the perimeter of a rectangle increases, the area also increases". She shows an example from several she has done:

$\mathbf{P = 16\,cm}$	$\mathbf{P = 24\,cm}$
$\mathbf{A = 16\,cm^2}$	$\mathbf{A = 32\,cm^2}$

   (e) Is the theorem true or false? Justify your response in a mathematical way.

   (f) List 1 misconception students may have when given this scenario.

Section B

2. Prove: if $\lim_{x \to a} f(x) = 0$ and $g(x)$ is bounded, then $\lim_{x \to a} f(x)g(x) = 0$.

3. The linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is defined by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 \phantom{{}+x_3} + 3x_4 \\ -x_1 + x_2 + x_3 + x_4 \\ 4x_1 + 5x_2 - x_3 + 5x_4 \\ -5x_1 - x_2 + 3x_3 + 6x_4 \end{pmatrix}$$

   (a) Find a basis for the kernel $\{v \in \mathbb{R}^4 : Tv = 0\}$ of T.

   (b) Find a basis for the range $\{Tv : v \in \mathbb{R}^4\}$ of T.

4. Consider the subgroup H of S_4 generated by the permutation $(1\ 2\ 3\ 4)$.

   (a) How many left cosets of H are there in S_4?

   (b) Is H a normal subgroup of S_4?

5. The random variable X has a Poisson distribution with mean μ, that is

$$\mathbb{P}(X = k) = e^{-\mu} \frac{\mu^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

   (a) Find the rth factorial moment

$$\mathbb{E}(X(X-1)(X-2)\cdots(X-r+1)).$$

   (b) Find $\mathrm{Var}(X)$.