Mathematics Research Day

April 26, 2018
1:00pm–3:00pm
Dunn Hall 249

Vanishing Relaxation Time Dynamics of the Jordan Moore-Gibson-Thompson (JMGT) Equation arising in High Frequency Ultrasound (HFU)

Sutthirut Charoenphon

The (third-order in time) JMGT equation is a nonlinear (quasilinear) Partial Differential Equation (PDE) model introduced to describe the acoustic velocity potential in ultrasound wave propagation. One begins with the parabolic Westervelt equation governing the dynamics of the pressure in nonlinear acoustic waves. In its derivation from constitutive laws, one then replaces the Fourier law with the Maxwell–Cattaneo law, to avoid *the paradox of the infinite speed of propagation*. This process then gives raise to a new third time derivative term, with a small constant coefficient τ, referred to as relaxation time. As a consequence, the mathematical structure of the underlying model changes drastically from the parabolic character of the Westervelt model (whose linear part generates a s.c, analytic semigroup) to the hyperbolic-like character of the JMGT model (whose linear part generates a s.c, group on a suitable function space). It is therefore of both mathematical and physical interest to analyze the asymptotic behavior of hyperbolic solutions of the JMGT model as the relaxation parameter $\tau \geq 0$ tends to zero. In particular, it will be shown that for suitably calibrated initial data one obtains at the limit exponentially time-decaying solutions. The rate of convergence allows one then to estimate the relaxation time needed for the signal to reach the target. The interest in studying this type of problems is motivated by a large array of applications arising in engineering and medical sciences. These include applications to welding, lithotripsy, ultrasound technology, noninvasive treatment of kidney stones.

Excursions into Controllability of a Chemotaxis System via Diffusive Phenomena

Stephen Guffey

In this talk we introduce a PDE model for a bacterial infection in a chronic wound. One particularly interesting feature of the model, mathematically speaking, is a chemotactic reaction between two of the unknowns. Said reaction is modeled after the Keller-Segel model for chemotaxis. We introduce concepts of controllability of the model from a functional analytic viewpoint, which brings a so-called observability condition. To motivate a discussion of common methods

for obtaining the observability condition (primarily via Carleman estimates), we first investigate how such methods can be applied in the simplified case of a coupled system of diffusion equations. During the course of the talk we will note particular difficulties and limitations for our system that arise along the way.

Topological properties of operations on spaces of continuous functions and integrable functions

Holly Renaud

The classical open mapping theorem states that every continuous and linear surjective map between two Banach spaces is open. However, an example proposed by W. Rudin shows that this property does not extend to bilinear maps. In this talk, we present several results that deal with different types of maps between normed spaces.

Highly linked tournaments with large minimum out-degree

Richard Snyder

A directed graph is said to be k-linked if for any two disjoint sets of k vertices $\{x_1, \ldots, x_k\}$ and $\{y_1, \ldots, y_k\}$, there exist vertex disjoint directed paths $P_1, \ldots, P_k$ such that P_i goes from x_i to y_i for each $i \in \{1, \ldots, k\}$. Graph linkedness is a very natural extension of the usual concept of graph connectedness. In this talk, we shall examine the relationship between these two notions in tournaments (those directed graphs obtained from orienting the edges of a complete graph). In particular, we show that there exists a function $f\colon \mathbb{N} \to \mathbb{N}$ such that for any positive integer k, if T is a strongly $4k$-connected tournament with minimum out-degree at least $f(k)$, then T is k-linked. This resolves a conjecture of Pokrovskiy up to a factor of 2 of the required connectivity. Along the way, we show that a tournament with sufficiently large minimum out-degree contains a subdivision of a complete directed graph. This is based on joint work with António Girão.