Recent Trends in Operator Theory and Applications

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Plenary Talks

Abstracts

From Toeplitz matrices to black holes, and beyond

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What do Toeplitz matrices, random matrix models, orthogonal polynomials, Painlev transcendents, the KdV equation, and black holes, seemingly very unrelated subjects, have in common? These, and a variety of other mathematical problems, can be studied by means of the so called Riemann-Hilbert method. In this talk we briefly describe what a Riemann-Hilbert problem is and present several recent applications, from the spectral properties of Toeplitz operators to exact solutions of Einstein field equations.

Multipliers and equivalences between Toeplitz kernels

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In this talk we review some classical and more recent results concerning kernels of Toeplitz operators in Hardy spaces. In particular we focus on the existence of so-called maximal vectors, which determine the kernel in a precise sense, and on multipliers between kernels of Toeplitz operators. It turns out that these multipliers can be characterized in terms of certain test functions, which are precisely the maximal vectors. Applications to model spaces, which are themselves Toeplitz kernels of a special kind, are presented.
Joint work with Jonathan Partington.

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Hermitian operators and isometries on Banach algebras of continuous maps with values in unital C^*-algebras

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We consider Banach algebras of continuous maps whose values are continuous functions, matrices, or operators. In particular, we study Banch algebras; $\mathrm{Lip}(X,E)$, $\mathrm{lip}_\alpha(X,E)$ for a compact Hausdorff space X, $C^1(K,E)$ for $K=[0,1]$ or the unit circle, $\overline{A\otimes E}$ for a uniform algebra A, where E is a unital C^*-algebra such as the algebra of all continuous functions on a compact Hausdorff space, the algebra of matrices or operators on a Hilbert space. We study Hermitian operators and isometries on these Banach algebras. We show what is known and what is unknown.
This is a joint work with Shiho Oi.

On isometries on some Banach spaces - something old, something new, something borrowed, something blue: Part I and Part II

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Isometries are maps between metric spaces which preserve distance between elements. Although the beginnings of the study of isometries between Banach spaces coincide to the very beginnings of the Banach space theory, this area of research is still very active. One of the main problems is to give explicit description of isometries on a particular space. However, different types of spaces require different methods of approaching this problem. In these two talks, isometries on various important Banach spaces (both finite and infinite dimensional, both real and complex) will be described. Basic introductory material will be combined with several recent results related to isometries, including those on generalized bicircular and n-circular projections, roots of isometries, preservers of isometries etc. These recent results are from joint works with several colleagues: Bojan Kuzma, Chi-Kwong Li, Chih-Neng Liu, Edward Poon, Ngai-Ching Wong, etc.

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Compression and compact perturbation of operators: Part I and Part II

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Let $\mathbf{A} = (A_1, \ldots, A_m)$ be an m-tuple of bounded linear operators acting on a Hilbert space $\mathcal{H}$. The joint (p, q)-matricial range $\Lambda_{p,q}(\mathbf{A})$ is the set of m-tuple of $q \times q$ matrices $\mathbf{B} = (B_1, \ldots, B_m)$ such that $I_p \otimes B_j$ is a compression of A_j on a pq-dimensional subspace. This definition arises from the study of noiseless subsystem of quantum channels and covers various kinds of generalized numerical ranges for different values of p, q, m. In this talk, we discuss some recent results on $\Lambda_{p,q}(\mathbf{A})$. If $\dim \mathcal{H}$ is infinite, the definition of $\Lambda_{p,q}(\mathbf{A})$ is extended to $\Lambda_{\infty,q}(\mathbf{A})$ consisting of $(B_1, \ldots, B_m) \in \mathbf{M}_q^m$ such that $I_\infty \otimes B_j$ is a compression of A_j on a closed subspace of $\mathcal{H}$, and also the joint essential (p, q)-matricial range

$$\Lambda_{p,q}^{ess}(\mathbf{A}) = \bigcap \{\mathbf{cl}(\Lambda_{p,q}(A_1 + F_1, \ldots, A_m + F_m)) : F_1, \ldots, F_m \text{ are compact operators}\}.$$

Both sets are shown to be convex, and the latter one is always non-empty and compact. Furthermore, the results and techniques are extended to study of compression and perturbation of other classes of operators with special structure.

Invitation to linear preserver problems: Part I and Part II

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These talks survey results around two major axis. The first one concerns the Kaplansky problem; the history of the problem and several results are presented. The second one concerns some new preserver problems (concerning the generalized inverse, Fredholm or semi-Fredholm operators...). The common point of these results is that they are interesting only in the infinite dimensional situation. Several open questions are mentioned.

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An introduction to the geometry of spaces of operators on Banach spaces

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For Banach spaces X, Y we consider the space of bounded linear operators $\mathcal{L}(X, Y)$ and the space of compact operators $\mathcal{K}(X, Y)$. We give some sufficient conditions under which the space of compact operators is a strict ideal in the space of bounded operators. As an application, when $Y = C(K)$ we discuss the open problem, when is the bidual of $\mathcal{K}(X, C(K))$, a space of operators, valued in a continuous function space?

When are adjoints of operators very smooth points?

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For a non-reflexive Banach space X, $x_0 \in X$ is said to be a very smooth point, if there is a unique unit vector $x_0^* \in X^{***}$ such that $x_0^*(x_0) = \|x_0\|$. In this talk we give sufficient conditions for a $T \in \mathcal{K}(X, Y)$ so that $T^* \in \mathcal{K}(Y^*, X^*)$ is a very smooth point.

Invited Talks
Abstracts

On small combination of slices points in Banach spaces

Sudeshna Basu, *George Washington University, Washington D.C.*
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Small Combination of Slices (SCS) in the unit ball of a Banach space was first introduced by Godefroy et al, and subsequently analysed in detail in Scachermeyer and Rosenthal In this work, we introduce Ball- SCSP and Span- SCSP a geometric property of Banach space in terms of w^*-SCS points. We explore its relation with other geometric properties. We study certain stability results for leading to a discussion on in the context of ideals of Banach Spaces and spaces of operators. We study these properties in in C(K,X)-spaces and L(X,Y) for Y= C(K). We also study these properties in the context of tensor products of Banach spaces.
Part of this talk is based on joint work with T.S.S.R.K.Rao.

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Phase retrieval versus phaseless reconstruction

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In many engineering applications signals pass through linear systems which distort (or erase) phase information. Phaseless reconstruction is the problem of recovering a signal from a set of intensity measurements. In 2006, Balan/Casazza/Edidin introduced the frame theoretic study of phaseless reconstruction. Over the years, many people have replaced the term phaseless reconstruction with phase retrieval. Casazza asked if recovering the phase of a given vector was really equivalent to phaseless reconstruction. In this talk, we will show that phase retrieval is equivalent to phaseless reconstruction for both vectors and projections.
Joint work with Peter G.Casazza, Hanh Van Nguyen and Janet C.Tremain.

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Algebraic reflexivity on spaces of analytic functions

Priyadarshi Dey, *University of Memphis, Memphis, TN*
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We call a Banach space X algebraic reflexive if any locally surjective isometry on X is surjective. In this talk I will show that the space S^p is algebraically reflexive for any $p \neq 2$.

Pairs of operators coinciding on the orthocomplement of the sum of kernels
Marko Djikić, *Faculty of Sciences and Mathematics, University of Niš, Serbia*
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We will demonstrate several results regarding pairs of operators acting on a Hilbert space in such a way that they coincide on the orthogonal complement of the sum of their kernels. Such operators include some well studied classes, the most prominent being the pairs of orthogonal projections. The results presented will include geometrical aspects (closedness of ranges, sum of ranges, etc.) and algebraical aspects (properties of linear combinations, pseudo-inverses, etc.).

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Approximations by discrete operators
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In this talk, we will discuss the approximation properties of discrete versions of Picard, Gauss-Weierstrass, and Poisson-Cauchy singular operators. We will describe not only quantitatively the point-wise and uniform convergences but also statistical convergence of these operators to the unit operator by involving the modulus of smoothness of a uniformly continuous function.

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Spear vectors on Banach spaces
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Spear Vectors: Let X be a Banach space. An element $z \in S_X$ is spear (or spear vector) if

$$||z + \mathbb{T}x|| = 1 + ||x|| \text{ for every } x \in X$$

The main purpose of this talk is to show the solution to the following open problems:
Problem 1. Let X be a Banach space. If $Spear(X)$ is not compact does X contain a copy of c_0 or l_1?
Problem 2. Let X be a smooth Banach space and $Spear(X^*) \neq \phi$, can we deduce that $X \cong \mathbb{C}$?
These problems are proposed as open problems at the end of the book *Spear operators between Banach spaces* by Vladimir Kadets, Miguel Martín, Javier Merí and Antonio Pérez . We will develop and discuss tools and material needed for these problems. To this end, we will see some important properties of spear vectors and will discuss about spear vectors of some common Banach spaces.

Bases in the space of regular multilinear operators on Banach lattices
Khazhak Navoyan, *The University of Mississippi, Oxford, MS*
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For Banach lattices $E_1, \cdots, E_m$ and F with 1-unconditional bases, we show that the monomial sequence forms a 1-unconditional basis of $\mathcal{L}^r(E_1, \cdots, E_m; F)$, the Banach lattice of all regular m-linear operators from $E_1 \times \cdots \times E_m$ to F, if and only if each basis of $E_1, \cdots, E_m$ is shrinking and every positive m-linear operator from $E_1 \times \cdots \times E_m$ to F is weakly sequentially continuous. As a consequence, we obtain necessary and sufficient conditions for which the m-fold Fremlin projective tensor product $E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_m$ (resp. the m-fold positive injective tensor product $E_1 \check{\otimes}_{|\epsilon|} \cdots \check{\otimes}_{|\epsilon|} E_m$) has a shrinking basis or a boundedly complete basis.
Joint work with Donghai Ji, and Qingying Bu.

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Best approximation in normed linear spaces
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Best approximation in normed linear spaces is a well studied subject and it has numerous applications in many other streams in analysis. The theory have some nice interplay with the geometry of the unit ball and the differentiability of the dual norm. If the best approximation is guaranteed for a given subspace it is natural to study how this set of best approximation is located. Various strengthenings of best approximation comes in this context. Instead of finding best approximation from a given point to a linear subspace one can generalize this notion for a given closed and bounded subset from the same linear subspace and hence the center of a arbitrary closed bounded set can be defined. Similar to the continuity of metric projection the continuity of Chebyshev center is also of special interest. I will try to discuss about these topics, related well known results and some recent observations.

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Entanglement in single-shot quantum channel discrimination, and norms on linear maps of matrices
Daniel Puzzuoli, *Institute for Quantum Computing, Waterloo, Canada*
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A fundamental task in quantum information is to determine which quantum channel, from a given set, is acting on a system. We will discuss 'single-shot quantum channel discrimination': a particular version of this task in which the unknown channel is known to be drawn from one of two channels, and the channel may only be used once. It is well-known that entanglement can provide an advantage in this task, and it is natural to investigate how large this advantage can be. We will discuss results and open problems motivated by this question. Mathematically, this requires understanding the largest possible gaps between various norms on linear maps of matrices. For example, if arbitrary entanglement is allowed, the optimal performance in this task is given as an expression involving the completely bounded norm. One of our results is that matrix transposition is, essentially, the unique linear map on M_n with maximal gap between its norm and its completely bounded norm. (Results discussed may be found in Quantum 2, 51 (2018).)

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Topological properties of operations on spaces of continuous functions and integrable functions
Holly Renaud, *University of Memphis, Memphis, TN*
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The classical open mapping theorem states that every continuous and linear surjective map between two Banach spaces is open. However, an example proposed by W. Rudin shows that this property does not extend to bilinear maps. In this talk we present several results that deal with different types of openness of maps between normed spaces.

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Using linear functionals to study nonlinear functionals and their support sets
Jessica Stovall, *University of North Alabama, Florence, AL*
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Any Dedekind complete Banach lattice E with a quasi-interior point e is lattice isomor- phic to a space of continuous, extended real-valued functions defined on a compact Hausdorff space X. An orthogonally additive, continuous, monotonic, and subhomogeneous nonlinear functional $T : E \to \mathbb{R}$ is studied. In this case, the concept of integration is no longer valid. In a paper with William A. Feldman (University of Arkansas) a measure $\overline{\mu}$ related to the nonlinear operator T is constructed and the linear operator associated with $\overline{\mu}$ is studied. This talk discusses these results and demonstrates how these results can be used to study nonlinear functionals. Additional results regarding support sets will be presented.