

## Nonlinear Wave Phenomena, Math 7501/8501, Fall 2017

Professor: Hongqiu CHEN

Class hours: MWF: 10:20-11:15am at DH203

Office hours: Wed: 1:00pm-3:00pm or by appointment

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**Textbook** : **Linear and Nonlinear Waves**, G.B. Whitham, Wiley-interscience ISBN 0-471-94090-9

**Attendance** : Required. Be on time for class.

**Methods** : The course is lecture based.

**Abstract** : Model equations for waves that take account of both nonlinearity and dispersion have their genesis in the discovery of the solitary wave by John Scott Russell. The famous story of Scott Russells encounter with the solitary wave in 1834 has been retold many times.

The subject of the solitary wave generated much discussion in the 19<sup>th</sup> century at the hands of Airy, Stokes and later Boussinesq and Korteweg and de Vries. However, in the first half of the 20<sup>th</sup> century, the subject was quiet. This was to change in the early 1960's when the area came back to life with the discovery by Kruskal and Zabusky of the exact interaction of solitary waves. Since then, there has been and continues to be steadily increasing activity in this area, with at least three Field's Medalists having work on this subject cited.

The present course will be concerned with some of the more elementary aspects of nonlinear, dispersive waves. The discussion will initially center around the genesis of the area, which was theory of surface wave propagation. Once model equations for this phenomena are obtained, attention will be turned to mathematical issues thrown up by these models. This will involve us in the theory of partial differential equations and methods from functional analysis and numerical analysis and simulation.

Students should have a grasp of basic Fourier analysis and elementary real variables. Undergraduate partial differential equations will also be assumed, but much of the theory will be developed *ab initio*.

### *Table of Contents :*

1. Introduction and a brief review of the history: soliton phenomena and rediscovery of KdV equation in 1960s.
2. Derivation of various model equations for waves in dispersive media: Korteweg-de-Vries equation and a class of formally equivalent equations.
3. Mathematical theory for the initial-value problems: Bourgain spaces and Tao methods.
4. Solitary waves, their existence, stability and instability.
5. Initial-boundary-value problems and the inclusion of damping effects; bore propagation.
6. Applications to modeling of various physical systems.