MATH-7/8504– Fall 2017-
Analysis with applications to PDE's
Instructor: Irena Lasiecka, Lasiecka@memphis.edu.
Time: Tu-Th 3:50 -5:15
What is this course about?
The main aim of the course is to provide a background in Real and Functional Analysis suitable for studying more advanced topics in PDE's, Mathematical Physics and Calculus of Variations. One of the first theme is the theory of distributions. This will allow us to generalize the standard "calculus" to be operative on objects which do not have the required regularity properties [differentiability, integrability etc]. Once we learn how to differentiate, Fourier transform, multiply "weird" functions (called dstributions) we can then address questions related to the existence of "solutions" to a given PDE or minimization problems etc. The world "solution"is still undefined, as we need to find a "home" for our solutions. This "home" will be much larger than typically secured by the classical analysis-calculus. This will lead to a construction of various function spaces-Sobolev, Besov, Lizorkin, Hardy etc which will serve as a potential residence for solutions to our problems. Thus this part of the course will be devoted to studying Sobolev and related spaces along with their properties such as compactness, density,embeddings,traces etc. The third part of the course will be devoted to variational principles which provide a common framework for studying abstract PDE's, and, more generally, various extremal problems.

1.Theory of distributions.
- Spaces od distributionsD' and Schwartz spaces S'.
- Fourier's transform in Schwartz spaces.
- Calculus of distributions.

2. Sobolev spaces.
- Sobolev's saces in R^n and bounded domains.Localization.
- Sobolev's emebddings ,inequalities and compactness.
- Trace theorems .
- Triebel-Lizorkin spaces.

3.. Variational Analysis .
- Projection operators.
- Variational inequalities
- Weak solutions to PDE;s
- Regularity theory .

4. Applicatiosn to :
- Elliptic equations: scalar and systems- elasticity
- Stokes equations –fluids.
- Maxwell equations**.** –electromagnetism**.**

Text: S. Kesavan, Topics in Functional Analysis with Applications. J. Wiley, Chapters 1-3. H. Brezis, Functional Analysis, Sobolev Spaces and PDE's. Springer, 2011

Prerequisite: Real Analysis 7351- 7352. Functional Analysis desired but not required.

Grading Policy: Presentation in class and 4 graded homeworks. Work in groups is encouraged. However solutions need to be written down individually.