Math 7352: Ergodic theory course syllabus

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> ***COVID-19 note:*** *We start this class online, but if circumstances improve, we may have face to face classes. I will not make a decision about this without consulting with the students in this class. To read about our department's policy regarding the virus, please go to https://www.memphis.edu/msci/news/covid.php.*

In this course, I try give some basic ideas and results from ergodic theory which may benefit every mathematician: analysts, combinatorists, number theorists, probabilists. We'll see how focusing on and searching for measure preserving transformations, we discover unexpected connections among physics, probability, number theory, fourier analysis. The focus will be on presenting ideas and motivations instead of technologies. While ergodic theory is an abstraction of common ideas, I think the road to discovery and understanding is through specific examples, hence in this class, we'll discover general results through analyzing specific ones.

Notes I'll write my own notes for the course, but I'll use ideas from the notes of Green, Frantzikinakis, Furstenberg, Halmos, Kalikow, McCutcheon, Petersen.

Prerequisites I am not going to require the completion of any specific course since you might have covered real analysis elsewhere, but you need to know Lebegue's theory of integration and the basics of Hilbert spaces.

Expectations from students Students are expected to study the proofs presented in class, work out details, ponder, ask questions, catch errors, solve problems and possibly present their solutions in class, (One class each week will be devoted to problem solving.)

Topics to be covered

Origins in physics Phase space, Liouville's theorem, how to interpret a sequence of measurements on a physical system.

Laws of large numbers Weak and strong laws with simple proofs, based on the "subsequence" method.

Uniform distribution in the unit interval Weyl's fourier transform method and his metric theorem.

Mean ergodic theorem of von Neumann, ergodicity F. Riesz' simple proof even futher simplified, reducing the theorem to the Pythagorean theorem, von Neumann's original proof via the spectral theorem. Invariance, projection, conditional expectation.

Pointwise ergodic theorem Calderón's transference principle: how the Hardy-Littlewood maximal inequality implies the pointwise ergodic theorem. How a result for the simplest of all measure preserving transformations, the shift on the integers, implies the result for all measure preserving transformation. This is a big wow, and we'll hear about Bourgain's rediscovery of the transference principle, and its crucial role in his Fields medal worthy research.

A word about the Hilbert transform How the ergodic and usual Hilbert transforms are related, and their basic properties.

Modelling the integers in a dynamical system The Kakutani-Rokhlin's tower.

Mixing, weak mixing transformations Mixing in probability, coordinate shifts (Bernoulli systems), every measure preserving transformation splits into rotational and weak mixing parts, the general philosophy.

Recurrence theorem "Everything will happen again eventually" While the recurrence theorem is "only" the general formulation of the pigeon hole principle, I don't know of anything so simple with such wide applicability, even with philosophical consequences.

The correspondence principle of Furstenberg Modelling sets of integers of positive density on dynamical systems. Sophisticated recurrence theorems and their implications to number theory. Finding arithmetic progressions in integers with positive density.

Topological dynamics, recurrence Applications to multiples of irrational numbers.

Entropy I'd be shocked if I had time to cover this. Good and essential stuff, though.