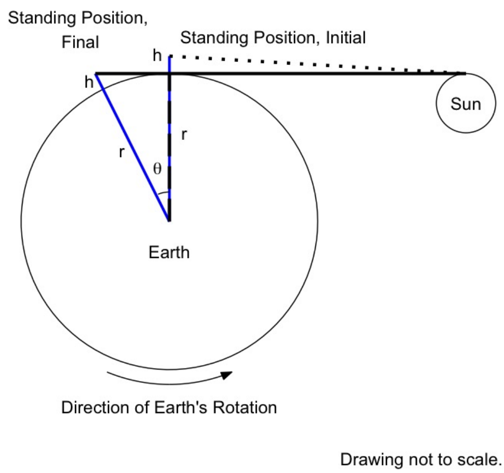
Estimating the Radius of the Earth from the Equator

The Earth's radius can be estimated with a stopwatch on a beach near the Equator in the following way. Determine (with suitable eye protection) when the sun drops below the horizon while reclining on the sand of the beach. Then start the stop watch and stand up. Stop the chronometer when the sun drops below the horizon while still standing. The ratio of the time on the stopwatch to the the length of a day is the same as the ratio of the angle through which the Earth rotated during that time period to it revolution of 360°(or 2π radians). See the picture below



This is analogous to the light of the Sun at sunset hitting the top of a building while there are already shadows on the lower part. The shadows, however, could be also due to other objects than the horizon of the Earth.

$$\frac{\text{Time Measured}}{\text{Length of a day}} = \frac{\text{Angle of Earth's Rotation}}{360°}$$

or

$$\theta(\text{in }^\circ) = \text{Angle of Earth's Rotation} = (360^\circ)\left(\frac{\text{Time Measured}}{\text{Length of a Day}}\right)$$

Notice that the time measured is most likely in seconds, so you should figure out how many seconds are in one day, or one entire revolution:

$$1 \text{ day} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = 86{,}400 \text{ seconds}$$

So now take the time that you measured and work out the angle that the Earth has rotated in this time period between the two observed sunsets.

Note: You could also obtain the angle in radians (although this is not necessary).

$$\theta(\text{in radians}) = \theta(\text{in }^\circ)\left(\frac{\pi}{180^\circ}\right)$$

Now, by hypothesis, you only have a stopwatch, but we also making the (reasonable?) assumption that you know or can estimate your height. More precisely, it is assumed that you can estimate or determine the separation between your eyes when reclining on the beach and when standing on the beach. It is somewhere around 5 or 6 feet for most people. If you have a driver's license, it is about a few inches less than your listed height.

Now you have a right triangle, where the line of sight from you eyes (while standing) to the sun at sunset is a right angle, with one leg being very nearly the radius of the earth, the hypotenuse being the sum of the radius of the Earth and the height of your eyes, h). The angle you determined above the adjacent to the leg given by the radius of the earth, so

$$\cos\theta = \frac{R_{\text{Earth}}}{R_{\text{Earth}} + h}$$

After some algebra,

$$R_{\text{Earth}} = \frac{h\cos\theta}{1 - \cos\theta}$$

If you measured the time to be 10.4 s and your the height of your eyes above your position while reclining is $h = 1.76$ m, what is your estimate of the radius of the Earth?

If you have any further questions, let us know.